Commun. Math. Phys. 146, 637–638 (1992)

Communications in Mathematical Physics

Erratum

Proof of Chiral Symmetry Breaking in Strongly Coupled Lattice Gauge Theory

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Received January 3, 1992

Commun. Math. Phys. **139**, 395 (1991)

The "constant" C_n in Theorem 3.11 still has a m-dependence; Theorem 3.11 has to be restated as:
For all $m \in \mathscr{W}$ there is a $\kappa(m) > 0$ such that for all $n \in \mathbb{N}$ and $x_1, \ldots, x_n \in \mathbb{Z}^\nu$,

$$|\langle \sigma^L \rangle^T| \leqq \tilde{C}_n B_n(m) e^{-\kappa(m)\vartheta(x_1, \ldots, x_n)}, \tag{1}$$

where $\tilde{C}_n$ depends only on n and m_0 (larger than the inverse radius of convergence of the cluster expansion), and

$$B_n(m) = \begin{cases} \max\{1, |\mathrm{Re}\, m|^{-n}\} & \text{for} \quad |m| \leqq m_0, \\ 1 & \text{for} \quad |m| > m_0. \end{cases} \tag{2}$$

The reason for this is an error in our original proof: the bound $|\langle \sigma^L \rangle| \leqq 1$ which we used holds only for real m. For general $m \in \mathscr{W}$ with $\mathrm{Re}\, m \neq 0$, the monomer-dimer results of Gruber and Kunz [1] imply only the weaker bound

$$|\langle \sigma^L \rangle| \leqq |\mathrm{Re}\, m|^{-|L|}. \tag{3}$$

To prove clustering from this, fix $m' > 0$ and change the definition of u_L to

$$u_L(m) = \frac{1}{\vartheta(L)} \log\left(\frac{|\langle \sigma^L \rangle^T|}{\tilde{C}(L) \max\{1, m'^{-|L|}\}} \right), \tag{4}$$

then $u_L(m) \leqq 0$ for all m with $|\mathrm{Re}\, m| > m'$, $u_L(m) < 0$ for $|m| > m_0$, and u_L is still subharmonic in m, so the Penrose-Lebowitz subharmonicity argument [2] implies $u(m) = \limsup u_L(m) < 0$ for all m with $|\mathrm{Re}\, m| > m'$, as worked out in our paper. Given m with $\mathrm{Re}\, m \neq 0$, Theorem 3.11, as stated above, is then obtained by taking e.g. $m' = \frac{1}{2}|\mathrm{Re}\, m|$. The proof also implies that $\kappa(m)$ is bounded below as $m \to 0$, $\kappa(m) \geqq M |\mathrm{Re}\, m|$ for $|m| \leqq m_0$, where M depends only on m_0.

For real $m \neq 0$ and the two-point function we can get rid of $B_2(m)$:

$$|\langle \sigma_0 \sigma_x \rangle - \langle \sigma_0 \rangle \langle \sigma_x \rangle| \leqq e^{-\kappa(m)|x|}, \tag{5}$$

where $|x| = \max_{i=1,\dots\nu} \{|x_i|\}$. This can be seen as follows: let $m \in \mathbb{R}$. Denoting $\mathscr{L}_+ = \{L \in \mathscr{L} : \operatorname{supp} L \subset \{x \in \mathbb{Z}^\nu : x_1 \geqq 0\}\}$, our physical Hilbert space is the space $\mathscr{H}$ of equivalence classes of sequences in $l^1(\mathscr{L}_+)$, with scalar product (cf. Definition 3.14)

$$\langle \psi, \chi \rangle = \sum_{L, L' \in \mathscr{L}_+} \bar{a}_L b_{L'} \langle \sigma^{L_r + L'} \rangle \tag{6}$$

for ψ, χ represented by sequences $(a_L)_{L \in \mathscr{L}}$ and $(b_L)_{L \in \mathscr{L}}$. Let $\mathscr{D}$ be the subspace formed by the sequences with only finitely many nonzero entries. We denote the transfer operator corresponding to translations by $2e_1$ by T_1, then $0 \leqq T_1 \leqq 1$ (second inequality from the boundedness at *real m*), and $T_1^* = T_1$, so the spectrum $\sigma(T_1) \subset [0, 1]$. By translation invariance, Ω, represented by $a_0 = 1$, $a_L = 0$ for $|L| > 0$, is an eigenvector of T_1 with eigenvalue 1. We denote the projection operator on Ω by P_Ω and also the spectral projection of T_1 on $I \subset [0, 1]$ by P_I.

By a similar argument as given above, one can show that for all $L, M \in \mathscr{L}$,

$$|\langle \sigma^L \sigma^M \rangle - \langle \sigma^L \rangle \langle \sigma^M \rangle| \leqq C(m, L, M)\, e^{-\kappa(m) d(L, M)}, \tag{7}$$

where $d(L, M)$ is the minimal distance between the supports of L and M, the function $B_n(m)$ enters in the prefactor $C(m, L, M)$ and $\kappa(m)$ does not depend on L and M. For all $\psi, \chi \in \mathscr{D}$ and for all $t \in \mathbb{N}$,

$$|\langle \psi, T_1^t \chi \rangle| \leqq \tilde{C}(m, \psi, \chi) e^{-\kappa(m) t}, \tag{8}$$

which implies that for all $f \in \mathscr{D} : P_{(e^{-\kappa}, 1)} f = 0$. Since $\mathscr{D}$ is dense in $\mathscr{H}$, $\sigma(T_1) \setminus \{1\} \subset [0, e^{-\kappa}]$. Let $x_i \in 2\mathbb{N}$, then

$$|\langle \sigma_0 \sigma_x \rangle - \langle \sigma_0 \rangle \langle \sigma_x \rangle| = \left\langle \eta, (T_1^{x_1} - P_\Omega) \prod_{i=2}^{\nu} T_i^{x_i} \eta \right\rangle \leqq \|\eta\|^2 e^{-\kappa(m) x_1} \tag{9}$$

(η is the state corresponding to σ_0 and T_i, $i \geqq 2$ are the unitary translations in spatial directions). Using the discrete rotational symmetry, we can repeat this argument for all other directions and get the bound. If x_i is not even for some i, the Schwarz inequality has to be applied once before proceeding with the given argument.

So, all our other results remain unchanged, because we only need clustering of the truncated two-point function in the form (5) for our proofs. Actually, for the lower bound for the chiral condensate the prefactor $B_n(m)$ does not play any role because we need clustering only at fixed $m > 0$ to identify the constant c in the formula for the Fourier-transformed two-point-function $\hat{T} = c\delta + \hat{g}$ as $c \langle \bar{\psi} \psi(x) \rangle^2$ and because the infrared bound for $\hat{g}$ is independent of the considerations about clustering. Only for the proof that the mass gap must go to zero as m vanishes, Remark 3.12 and the bound (4.19), we had to show that $B_2(m)$ is absent for real m.

References

1. Gruber, C., Kunz, H.: Commun. Math. Phys. **22**, 133 (1972)
2. Lebowitz, J., Penrose, O.: Commun. Math. Phys. **39**, 165 (1974)

Communicated by J. Fröhlich