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Asymptotic Expansion of the Logarithm of the Partition Function

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Abstract. A method is presented permitting one to find in principle all the non-decreasing terms of the asymptotic expansion of the logarithm of the partition function when the volume of region increases. The constructions are carried out at low activity for lattice systems with general n-body interactions, and continuous systems with two-body interactions.

1. Introduction

The aim of this paper is to study the asymptotic behaviour of the logarithm of a partition function $\ln \Xi(\Lambda)$ when the volume of the region Λ increases. Under natural assumptions about the grand canonical Gibbs ensemble and for appropriate classes of regions all the non-decreasing terms of the asymptotics are obtained in both cases of continuous and lattice systems.

The main term of this asymptotic (proportional to the volume $|\Lambda|$ of the region Λ) follows from the theorem by Lee and Yang [1]. For the lattice case the second term (proportional to the area of the boundary $\Gamma(\Lambda)$ of Λ] was obtained by Dobrushin under assumptions which provide the absence or the presence of a phase transition [2]. In the case of continuous systems the second term turns out to be proportional to the area of the boundary $\Gamma(\Lambda)$. The next terms depend on geometrical characteristics of $\Gamma(\Lambda)$.

The asymptotic expansion for the logarithm of the partition function is obtained in this paper as a special case of expansions for the integrals of the so-called clusterwise smooth translation-invariant functions over all finite subsets of the bounded region Λ. These asymptotic expansions are obtained for lattice systems with general n-body interactions and for continuous systems with two-body interactions.

2. Preliminaries

Consider a continuous system of particles in some bounded region Λ of the ν-dimensional Euclidean space $\mathbb{R}^\nu$ ($\nu \geqq 1$) interacting via pair potential $\Phi(x)$,

$x \in \mathbb{R}^\nu \setminus \{0\}$, where Φ is an even function. The space of states of the system is the space $C(\Lambda)$ of all finite subsets (configurations) of the region with a naturally defined σ-field $\mathscr{E}(\Lambda)$ and the Lebesgue measure dc (see [3, 4]). Analogously by $C(\mathbb{R}^\nu)$ we denote the space of all finite configurations $c \subset \mathbb{R}^\nu$ with the σ-field $\mathscr{E}(\mathbb{R}^\nu)$ and the Lebesgue measure, so that their restrictions to the space $C(\Lambda) \subset C(\mathbb{R}^\nu)$ coincide with the σ-field $\mathscr{E}(\Lambda)$ and the Lebesgue measure on $C(\Lambda)$ for any bounded region $\Lambda \subset \mathbb{R}^\nu$.

The Gibbs probability distribution on $C(\Lambda)$ is given by the density

$$p_{\Lambda,\beta,z}(c) = \frac{z^{N(c)} e^{-\beta U(c)}}{\Xi(\Lambda,\beta,z)} \tag{1}$$

with respect to the Lebesgue measure on $C(\Lambda)$. Here $N(c)$ is the number of points of c, $U(c) = \sum_{\{x,y\} \subset c,\, x \neq y} \Phi(x-y)$ is the energy of the configuration c, z (activity) and β (inverse temperature) are positive parameters, and finally

$$\Xi(\Lambda,\beta,z) = \int_{C(\Lambda)} z^{N(c)} e^{-\beta U(c)} dc$$

is the grand partition function.

In a similar way we shall consider the case of a lattice system with n-body interaction $\Phi(c)$, $c \in C(\mathbb{Z}^\nu)$, $\Phi(\emptyset) = 0$, where $C(\mathbb{Z}^\nu)$ is the set of all finite subsets of the ν-dimensional integer lattice $\mathbb{Z}^\nu$. In this case the probability of the configuration c is given by formula (1), where $\Lambda \subset \mathbb{Z}^\nu$ is any bounded set, $U(c) = \sum_{\bar{c} \subset c} \Phi(\bar{c})$, and

$$\Xi(\Lambda,\beta,z) = \sum_{c \subset \Lambda} z^{N(c)} e^{-\beta U(c)}.$$

It is well known that under general assumptions on the potential Φ and on the sequence of regions $\Lambda_1 \subset \Lambda_2 \subset \ldots$ the following asymptotic (Lee-Yang theorem [1]) holds

$$\ln \Xi(\Lambda_{k,\beta,z}) = c_0 |\Lambda_k| + o(|\Lambda_k|), \qquad k \to \infty, \tag{2}$$

where $c_0 = c_0(\Phi,\beta,z)$ is a constant depending only on the potential Φ and parameters β and z (Gibbs specific free energy). In this paper the next terms of the asymptotic (2) up to the constant term will be found.

3. Conditions on the Potential

In the case of continuous systems we consider potentials satisfying the following conditions.

1. *(Stability)* There exists a constant $B \geqq 0$ such that for any configuration

$$\sum_{\{x,y\} \subset c,\, x \neq y} \Phi(x-y) \geqq -BN(c).$$

The following conditions 2–4 (except the case $k=0$ in condition 4) concern to the smoothness of the potential.

2. The potential Φ is finite-valued everywhere except possibly the closure $\bar{G}_0$ of some open set $G_0 \subset \mathbb{R}^\nu$ with smooth boundary where $\Phi = +\infty$. For every $\beta > 0$ the function $e^{-\beta\Phi}$ is piecewise smooth on the space $\mathbb{R}^\nu$, with k^{th} degree of smoothness.

Let us introduce the functions

$$V_k^\beta(x) = \max_{|s| \leqq k} \left| \frac{d^s}{dx^s} (e^{-\beta\Phi(x)}) \right|,$$

where $s = (s_1, \ldots, s_\nu)$ is a multi-index, and let

$$g_k^\beta(x) = \max\{V_k^\beta(x), |e^{-\beta\Phi(x)} - 1|\}, \quad k \geqq 1; \quad g_0^\beta(x) = |e^{-\beta\Phi(x)} - 1|. \tag{3}$$

3. There exists a constant B_k^β such that for every finite set $c \subset \mathbb{R}^\nu$,

$$\prod_{x, y \in c;\, x \neq y} V_k^\beta(x - y) \leqq \exp(N(c) B_k^\beta).$$

4. There exists an Euclidean invariant metric δ_k^β in $\mathbb{R}^\nu$ such that

$$g_k^\beta(x) \leqq D_k^\beta \exp(-\delta_k^\beta(0, x)),$$

where

$$\int_{\mathbb{R}^\nu} \exp(-\tfrac{1}{2}\delta_k^\beta(0, x)) dx < \infty$$

and D_k^β are some constants.

Let us denote by $\mathscr{U}_k(\mathbb{R}^\nu)$ the class of potentials $\Phi(x)$, $x \in \mathbb{R}^\nu\{0\}$, satisfying conditions 1–4, and by $\mathscr{U}_k^p(\mathbb{R}^\nu)$ the class of potentials $\Phi \in \mathscr{U}_k(\mathbb{R}^\nu)$ such that the metric δ_k^β in condition 4 satisfies the inequality: $\delta_k^\beta(0, x) \geqq p \ln(1 + |x|)$ for some integer p.

Passing to the lattice case, we shall call the metric δ on $\mathbb{Z}^\nu$ symmetric if δ is invariant under all automorphisms of the group $\mathbb{Z}^\nu$. We shall consider potentials Φ satisfying the following conditions:

5. There exists a symmetric metric δ on $\mathbb{Z}^\nu$ such that

$$\sum_{x \in \mathbb{Z}^\nu} \exp(-\tfrac{1}{2}\delta(0, x)) < \infty,$$

and

$$\sum_{0 \in c \in C(\mathbb{Z}^\nu)} |\Phi(c)| e^{L_\delta(c)} < \infty,$$

where $L_\delta(c)$ is the shortest length with respect to the metric δ of all the trees (i.e. connected graphs without closed loops) constructed on the points of c. [For example $\delta(0, x) = \gamma|x|$, $\gamma > 0$; or $\delta(0, x) = d \ln(1 + \alpha|x|)$, $d > 2\nu$, $\alpha > 0$].

4. Main Results

For any bounded region $\Lambda \subset \mathbb{R}^\nu$ we denote by Λ_1 the region which is homothetic to Λ with $|\Lambda_1| = 1$. Let $\mathscr{B}_a^k$, $a > 0$, $k = 0, 1, 2, \ldots$ be the class of convex bounded regions $\Lambda \subset \mathbb{R}^\nu$ with $(k+2)$-smooth boundary $\Gamma = \Gamma(\Lambda)$ such that in the neighbourhood of each point $x \in \Gamma(\Lambda_1)$ in some Cartesian coordinate system $\Gamma(\Lambda_1)$ is given by equation $\eta = f(\xi)$, where $f(\xi)$ has partial derivatives, bounded uniformly in modulus by the constant a. (Evidently the condition $|\Lambda_1| = 1$ gives a bound on a from below, but it is not essential for us.)

Theorem 1. *Let $\Phi \in \mathscr{U}_{k+1}^{p}(\mathbb{R}^{\nu})$ with $p \geqq \nu+(k+1)^2(k+2)(\nu+2)$, and the activity z be sufficiently small. Then for any region $\Lambda \in \mathscr{B}_a^k$, $1 \leqq k \leqq \nu$, the following expansion holds:*

$$\ln \Xi(\Lambda, \beta, z) = c_0|\Lambda| + \sum_{\ell=1}^{k} c_\ell(\Lambda) + R_k(\Lambda). \tag{4}$$

Here $c_0 = c_0(\Phi, \beta, z)$ is a constant which will be explicitly written below (see (11)), the quantities $c_\ell(\Lambda) = c_\ell(\Lambda, \Phi, \beta, z)$ have the form

$$c_\ell(\Lambda) = \int_{\Gamma(\Lambda)} \langle n(x), x \rangle b_\ell(x; \Lambda, \Phi, \beta, z) d\sigma(x), \tag{5}$$

where $n(x)$ is a unit exterior normal vector at the point $x \in \Gamma(\Lambda)$, $\langle \cdot, \cdot \rangle$ is a scalar product in $\mathbb{R}^\nu$, $d\sigma$ is the element of $(\nu-1)$-dimensional area of $\Gamma(\Lambda)$, and the explicit form of coefficients b_ℓ is given by (16). In addition, the quantities $c_\ell(\Lambda)$ have the following properties: if the region $\tilde{\Lambda}$ is homothetic to Λ with the homothety coefficient λ ($\tilde{\Lambda} = \lambda\Lambda$), then

$$c_\ell(\tilde{\Lambda}, \Phi, \beta, z) = c_\ell(\Lambda, \Phi, \beta, z)\lambda^{\nu-\ell}, \quad \ell = 1, \ldots, \nu,$$

and

$$c_\nu(\tilde{\Lambda}, \Phi, \beta, z) = c_\nu(\Lambda, \Phi, \beta, z) \ln \lambda.$$

Finally the quantity $R_k(\Lambda) = R_k(\Lambda, \Phi, \beta, z)$ satisfies the following estimate

$$|R_k(\Lambda)| \leqq \begin{cases} C_k|\Lambda|^{(\nu-k-1)/\nu}, & k < \nu-1, \\ C_{\nu-1} \ln|\Lambda|, & k = \nu-1, \\ C_\nu, & k = \nu \end{cases} \tag{6}$$

with the constants $C_i = C_i(\Phi, \beta, z, a)$.

Remark. As it follows below from Theorem 2 in the case $\nu = 2$ the logarithmic term in (4) vanishes. Apparently the same is true also in the case $\nu > 2$, however we have no proof.

In the case of the spherically symmetric potential Φ the expressions for $c_\ell(\Lambda)-s$ in (4) can be simplified. For example we consider $c_1(\Lambda)$ and $c_2(\Lambda)$.

Theorem 2. *Let the conditions of Theorem 1 be satisfied and the potential Φ be spherically symmetric. Then*

$$c_1(\Lambda) = d_1 S(\Gamma(\Lambda)),$$

$$c_2(\Lambda) = \begin{cases} \dfrac{1}{\nu-2} d_2 M(\Gamma(\Lambda)), & \nu > 2, \\ 0, & \nu = 2, \end{cases}$$

where $d_i = d_i(\Phi, \beta, z)$, $i = 1, 2$ are constants given explicitly by (32), (33), $S(\Gamma(\Lambda))$ is the area of the boundary $\Gamma(\Lambda)$ and the quantity M is given by

$$M(\Gamma(\Lambda)) = \int_{\Gamma(\Lambda)} \langle n(x), x \rangle \sum_{1 \leqq i < j \leqq \nu-1} (\kappa_\Gamma)_i(x)(\kappa_\Gamma)_j(x) d\sigma(x), \tag{7}$$

where $(\kappa_\Gamma)_j(x)$, $j = 1, \ldots, \nu-1$ are the principal curvatures of the surface $\Gamma(\Lambda)$ at the point x.

As it follows from Steiner's formula for the volume of a parallel body [5] the quantity M does not depend on the choice of the origin of coordinate system.

Now consider the case of a region with piecewise smooth boundary. Let $E_L : \mathbb{R}^\nu \to \mathbb{R}^\nu$ be the homothety of the space $\mathbb{R}^\nu$ with the coefficient L. For any convex region $\Lambda \subset \mathbb{R}^\nu$ with a piecewise smooth boundary, consider the family of homothetic regions $\{\Lambda_L = E_L \Lambda, L \geqq 1\}$ generated by Λ.

Theorem 3. *Let the spherically symmetric potential $\Phi \in \mathcal{U}_2^p(\mathbb{R}^\nu)$, $p \geqq 50$, and the activity z be sufficiently small. Then for the family of regions mentioned above the following expansion is valid*

$$\ln \Xi(\Lambda_L, \beta, z) = c_0 |\Lambda_L| + d_1 S(\Gamma(\Lambda_L)) + R_1(\Lambda_L),$$

where $c_0 = c_0(\Phi, \beta, z)$ and $d_1 = d_1(\Phi, \beta, z)$ are constants defined by formulae (11), (32), *respectively, and the quantity $R_1(\Lambda_L) = R_1(\Lambda_L, \Phi, \beta, z)$ satisfies the estimate*

$$|R_1(\Lambda_L)| \leqq C L^\alpha \tag{8}$$

for some $\alpha < \nu - 1$, where the constant $C = C(\Lambda, \Phi, \beta, z)$ depends on the initial region Λ.

Remark. One can choose a unique constant C in (8) for any suitably selected class of regions with piecewise smooth boundaries.

In the special case of a convex polyhedron, using another method we shall find explicitly all the terms of the asymptotic expansion including the constant term (see Theorem 4 below). The first method does not allow one to find the constant term explicitly.

For any convex polyhedron $\Lambda \subset \mathbb{R}^\nu$, denote by $\Lambda^{(i)}$, $i = 0, 1, \ldots, \nu - 1$, the family of all i-faces of the polyhedron Λ and by $|\lambda|$, $\lambda \in \Lambda^{(i)}$, the i-dimensional volume of λ. Define

$$\sigma(\Lambda) = \min\{\varrho(\lambda, \lambda') : \lambda \in \Lambda^0, \lambda' \in \Lambda^{(\nu-1)}, \lambda \notin \lambda'\},$$

where $\varrho(\lambda, \lambda')$ is the distance between the vertex λ and the face λ'. Let $\mathcal{C}(r, r', n_0)$ be the class of all convex polyhedrons satisfying the conditions:

(1) $\sigma(\Lambda) \geqq r |\Lambda|^{1/\nu}$,

(2) $\operatorname{diam} \Lambda \leqq r' |\Lambda|^{1/\nu}$,

(3) $\operatorname{card} \Lambda^{(\nu-1)} \leqq n_0$,

where $r, r' > 0$ and n_0 is an integer.

Theorem 4. *Let the spherically symmetric potential $\Phi \in \mathcal{U}_0^p(\mathbb{R}^\nu)$, $p \geqq 2\nu + 1$, and the activity z be sufficiently small. Then for any $\Lambda \in \mathcal{C}(r, r', n_0)$ the following expansion holds:*

$$\ln \Xi(\Lambda, \beta, z) = c_0 |\Lambda| + c_1 \sum_{\lambda \in \Lambda^{(\nu-1)}} |\lambda| + \ldots + c_{\nu-1} \sum_{\lambda \in \Lambda^{(1)}} |\lambda| + c_\nu + R(\Lambda), \tag{9}$$

where $c_0 = c_0(\Phi, \beta, z)$ is the same constant as above, c_i, $i = 1, \ldots, \nu$ are constants depending on the potential Φ and parameters β, z, while c_ν also depends on the set of all angles of the polyhedron Λ. Finally the quantity $R(\Lambda) = R(\Lambda, \Phi, \beta, z)$ satisfies the estimate

$$|R(\Lambda)| \leqq C |\Lambda|^{-1/\nu}$$

with the constant $C = C(\Phi, \beta, z, r, r', n_0)$.

Now consider the case of lattice systems. Let $\mathscr{C}(r)$, $r>0$, be the class of parallelepipeds $\Lambda\subset\mathbb{Z}^\nu$ satisfying the condition: $\operatorname{diam}\Lambda\leqq r|\Lambda|^{1/\nu}$.

Theorem 5. *Let the activity z be sufficiently small and the potential Φ satisfies condition 5. Then for any $\Lambda\in\mathscr{C}(r)$ the following expansion holds:*

$$\ln\Xi(\Lambda,\beta,z)=a_0|\Lambda|+a_1\sum_{\lambda\in\Lambda^{(\nu-1)}}|\lambda|+\ldots+a_{\nu-1}\sum_{\lambda\in\Lambda^{(1)}}|\lambda|+a_\nu+R(\Lambda),$$

where a_i, $i=0,1,\ldots,\nu$ are constants depending on the potential Φ and parameters β, z, the quantity $R(\Lambda)=R(\Lambda,\Phi,\beta,z)$ satisfies the estimate: $|R(\Lambda)|\leqq C|\Lambda|^{-1/\nu}$, with the constant $C=C(\Phi,\beta,z,r)$ while the remaining notations have the same meaning as in Theorem 4.

5. Strong Cluster Estimates. Cluster and Clusterwise Smooth Functions

The proofs of Theorems 1–5 are based on the strong cluster estimates of Ursell functions, truncated correlation (group) functions [6, 7] and their derivatives [8]. We describe the corresponding techniques below.

Let $\mathscr{M}_+^p$, $p>\nu$, be the class of functions on $\mathbb{R}^\nu$ defined by the formula: $f(x)=f_+(|x|)(1+|x|)^{-p}$, where f_+ is any non-negative bounded integrable function on $[0,+\infty)$. Define

$$\mathscr{N}_m^p=\left\{q\in\mathscr{M}_+^p:\int_{\mathbb{R}^\nu}q(x)dx<\frac{1}{m},\ \sup_x q(x)<2^{-m}\right\},\qquad m=1,2,\ldots.$$

Lemma 1 (see [8]). *Let $q\in\mathscr{N}_1^p$. Then the equation*

$$\int_{\mathbb{R}^\nu}f(y)q(x-y)dy=f(x)-q(y)$$

has the unique solution $f\equiv Vq$. Moreover, $Vq\in\mathscr{M}_+^p$ and

$$\int_{\mathbb{R}^\nu}(Vq)(x)dx=\int_{\mathbb{R}^\nu}q(x)dx\left(1-\int_{\mathbb{R}^\nu}q(x)dx\right)^{-1}.$$

Corollary. *Let $q\in\mathscr{N}_n^p$. Then the functions $V^kq\in\mathscr{M}_+^p$, $k=1,\ldots,n$, and the following equalities hold:*

$$\int_{\mathbb{R}^\nu}(V^kq)(x)dx=\int_{\mathbb{R}^\nu}q(x)dx\left(1-k\int_{\mathbb{R}^\nu}q(x)dx\right)^{-1}.$$

For any $q\in\mathscr{M}_+^p$ and $A>0$ we denote by $\mathscr{K}_{q,A}^p$ the class of functions $\psi(c)$, $c\in C_+(\mathbb{R}^\nu)=C(R^\nu)\backslash\{\emptyset\}$, satisfying the estimate

$$|\psi(c)|\leqq A\sum_{\gamma\in\mathscr{L}_c}\prod_{(x,y)\in\gamma}q(x-y),\tag{10}$$

where $\mathscr{L}_c$ is the set of all chains constructed on c. The estimate (10) we shall call the *strong cluster estimate* and the functions of the class $\mathscr{K}_{q,A}^p$ we shall call *cluster functions.*

For any region $\Lambda\subset\mathbb{R}^\nu$ define an operator H_Λ acting on the set $\mathscr{K}_{q,A}^p$, $q\in\mathscr{N}_1^p$, by the formula:

$$(H_\Lambda\psi)(c)=\int_{C_+(\Lambda)}\psi(c\cup\bar{c})d\bar{c},\qquad c\in C_+(\mathbb{R}^\nu).$$

Lemma 2 [9]. *Let* $\psi \in \mathscr{K}^p_{q,A_0}$, $q \in \mathscr{N}^p_n$. *Then for any regions* $\Lambda_1, \ldots, \Lambda_n \subset \mathbb{R}^\nu$:
1) *the function* $H_{\Lambda_1} \cdot \ldots \cdot H_{\Lambda_n}\psi \in \mathscr{K}^p_{V^n q, A_n}$, *where*

$$A_n = A_0 \prod_{k=0}^{n-1} \left[1 - \int_{\mathbb{R}^\nu} q(x)dx \left(1 - k \int_{\mathbb{R}^\nu} q(x)dx\right)^{-1}\right]^{-2};$$

2) *for any* $x \in \mathbb{R}^\nu$

$$\int_{c \in C(\mathbb{R}^\nu): c \cap S'_x(r) \neq \emptyset} (H_{\Lambda_1} \cdot \ldots \cdot H_{\Lambda_n}|\psi|)(x \cup c)dc \leqq A_{n+1}\lambda_{n+1}(r),$$

where $S'_x(r) = \mathbb{R}^\nu \backslash S_x(r)$, $S_x(r)$ *is the ball of radius* r *about* x, A_n *is the constant defined above, and*

$$\lambda_{n+1}(r) = \int_{y \in \mathbb{R}^\nu: |y| \geqq r} (V^{n+1}q)(y)dy.$$

Corollary. *Let* $\psi \in \mathscr{K}^p_{q,A_0}$, $q \in \mathscr{N}^p_1$. *Then for any* $x \in \mathbb{R}^\nu$

$$c_0(x, \psi) = \int_{C(\mathbb{R}^\nu)} \frac{\psi(x \cup c)}{N(c) + 1} dc < A_1. \tag{11}$$

If in addition the function ψ *is translation-invariant then the quantity* $c_0(x, \psi) = c_0(\psi)$ *does not depend on* x.

Now note that the set $C^N(\mathbb{R}^\nu)$, $N = 1, 2, \ldots$ (N-point configurations in $\mathbb{R}^\nu$) is naturally endowed with the structure of an $(\nu \times N)$-dimensional smooth manifold. A function $\psi(c)$, $c \in C_+(\mathbb{R}^\nu)$ will be called k-smooth if all the functions $\psi_N(c)$, $N = 1, 2, \ldots$ are k-smooth, where ψ_N is the restriction of ψ to the subspace $C^N(\mathbb{R}^\nu)$.

Finally, for any integer k, $A > 0$, and $q \in \mathscr{M}^p_+$, $p > \nu$, we denote by $\mathscr{K}^p_{k,q,A}$ the class of k-smooth functions $\psi(c)$, $c \in C_+(\mathbb{R}^\nu)$ all partial derivatives of which belong to the class $\mathscr{K}^p_{q,A}$. The functions of the class $\mathscr{K}^p_{k,q,A}$ we will call *clusterwise-k-smooth functions.*

Theorem 6 [8]. *If the potential* $\Phi \in \mathscr{U}^p_k(\mathbb{R}^\nu)$, *then the corresponding Ursell function and truncated correlation function are clusterwise-k-smooth.*

We have the following analogy to Lemma 2.

Lemma 3. [9]. *Let* $\psi \in \mathscr{K}^p_{k,q,A_0}$, $q \in \mathscr{N}^p_n$. *Then for any regions* $\Lambda_1, \ldots, \Lambda_n \subset \mathbb{R}^\nu$:
1) *the function* $H_{\Lambda_1} \cdot \ldots \cdot H_{\Lambda_n}\psi \in \mathscr{K}^p_{k,V^n q, A_n}$;
2) *for any* $x \in \mathbb{R}^\nu \Lambda$,

$$\int_{c \in C(\mathbb{R}^\nu): c \cap S'_x(r) \neq \emptyset} |(D^{n_c}H_{\Lambda_1} \cdot \ldots \cdot H_{\Lambda_n}\psi)(x \cup c)|dc \leqq A_{n+1}\lambda_{n+1}(r),$$

where $n_c = \{n(x); x \in c\}$ *is a multi-index with* $|n_c| = \sum_{x \in c} n(x) = k$ *and* D^{n_c} *is the symbol of differentiation (for details see* [8]*).*

Remark. For the lattice case by the techniques used above and by the strong cluster estimate of the truncated correlation function [6] one can define the class $\mathscr{K}^p_{q,A}$ of cluster functions with the same properties as in the case of continuous systems.

6. The Expansion for the Regions with Smooth Boundary

As was mentioned in the Introduction we shall consider a more general problem. It is known [9], that

$$\ln \Xi(\Lambda, \beta, z) = \int_{C(\Lambda)} \psi_{\Phi,\beta,z}(c)dc\,, \tag{12}$$

where $\psi_{\Phi,\beta,z}$ is the Ursell function corresponding to the potential Φ and parameters β and z. This representation suggests a natural useful generalization. Let

$$Q(\Lambda, \psi) = \int_{C(\Lambda)} \psi(c)dc\,,$$

where ψ is an arbitrary cluster function.

In this paper we shall investigate the asymptotic behaviour of the quantity $Q(\Lambda, \psi)$, when $\Lambda \to \infty$, for various classes of regions. The expansions for $\ln \Xi(\Lambda)$ (Theorems 1–5) will be special cases of the corresponding expansions for $Q(\Lambda, \psi)$ when $\psi = \psi_{\Phi,\beta,z}$.

Let for any bounded convex region $\Lambda \subset \mathbb{R}^\nu$, $V(\Lambda) = \{x \in \Lambda : \varrho(x, \Gamma(\Lambda)) < 1\}$, where $\varrho(x, \Gamma(\Lambda))$ is the distance between the point x and the boundary $\Gamma(\Lambda)$.

Theorem 7. *Let the translation-invariant function* $\psi \in \mathscr{K}^p_{q,A_0}$, $q \in \mathscr{N}^p_1$, $p \geqq 2\nu + 1$. *Then for any bounded convex region* $\Lambda \subset \mathbb{R}^\nu$, $Q(\Lambda, \psi) = c_0(\psi)|\Lambda| + R(\Lambda)$, *where* $c_0(\psi)$ *is defined by formula* (11) *and* $|R(\Lambda)| \leqq C(A_0, q)|V(\Lambda)|$.

Proof. We have

$$Q(\Lambda, \psi) = \int_\Lambda dx \int_{C(\mathbb{R}^\nu)} \frac{\psi(x \cup c)}{N(c)+1} dc + R(\Lambda) = c_0(\psi)\,|\Lambda| + R(\Lambda)\,,$$

where

$$R(\Lambda) = -\int_\Lambda dx \int_{c \in C(\mathbb{R}^\nu) : c \cap \Lambda' \neq \emptyset} \frac{\psi(x \cup c)}{N(c)+1} dc\,, \qquad \Lambda' = \mathbb{R}^\nu \backslash \Lambda\,.$$

Using Lemma 2 we obtain

$$|R(\Lambda)| \leqq A_1 \int_{V(\Lambda)} \lambda_1(\varrho(x, \Gamma(\Lambda)))dx + A_1 \int_{\Lambda \backslash V(\Lambda)} \lambda_1(\varrho(x, \Gamma(\Lambda)))dx\,.$$

Now from the monotonicity of λ_1, it follows that for any $x \in \Lambda \cap V(\Lambda)$ and $r = \varrho(x, \Gamma(\Lambda))$,

$$\lambda_1(r) = |V(\Lambda) \cap S_x(r)|^{-1} \int_{V(\Lambda) \cap S_x(r)} \lambda_1(r)dy \leqq |S_x(\tfrac{1}{2})|^{-1} \int_{V(\Lambda)} \lambda_1(\varrho(x, y))dy\,.$$

Thus

$$\begin{aligned} |R(\Lambda)| &\leqq A_1 |V(\Lambda)| \lambda_1(0) + A_1 \int_{V(\Lambda)} dx \int_{\Lambda \backslash V(\Lambda)} \lambda_1(\varrho(x, y))dy \\ &\leqq A_1 \left(\lambda_1(0) + \int_0^\infty \lambda_1(r)dr \right) |V(\Lambda)|\,, \end{aligned}$$

which concludes the proof of Theorem 7.

Under additional assumptions concerning the function ψ and region Λ, we shall study the form of the remainder $R(\Lambda)$ below.

Next let us describe in detail the class of regions to be considered. Let $\Lambda \subset \mathbb{R}^\nu$ be a bounded convex region, the boundary $\Gamma = \Gamma(\Lambda)$ of which is $(k+2)$-smooth, $(\nu-1)$-dimensional submanifold in $\mathbb{R}^\nu$. For any $x \in \Gamma$ we denote by $T_\Gamma(x)$ the tangent hyperplane of Γ at x. Choose an orthogonal coordinate system $(\xi^{(1)}, \dots, \xi^{(\nu-1)}, \eta) \equiv (\xi, \eta)$ in $\mathbb{R}^\nu$ with the origin at x such that the axis η is directed along the interior normal of the surface Γ and axes $\xi^{(1)}, \dots, \xi^{(\nu-1)}$ lie in the hyperplane $T_\Gamma(x)$ along the direction of the principle curvatures $(\kappa_\Gamma)_i(x)$, $i = 1, \dots, \nu-1$ of the surface Γ at the point x. By $\mathscr{D}_\Gamma(x)$ we denote the set: $\mathscr{D}_\Gamma(x) = \{\xi \in T_\Gamma(x) : |\xi| < \delta_\Gamma\}$, where $\delta_\Gamma = \inf\limits_{x \in \Gamma} \left\{\left(2 \max\limits_i (\kappa_\Gamma)_i(x)\right)^{-1}\right\}$. Let Γ_x be the connected component of the set $\{(\xi, \eta) \in \Gamma : \xi \in \mathscr{D}_\Gamma(x)\}$ containing the point x. It is evident that for any $x \in \Gamma$, Γ_x is given by the equation: $\eta = f_{\Gamma,x}(\xi)$, $\xi \in \mathscr{D}_\Gamma(x)$, where $f_{\Gamma,x}$ is $(k+2)$-smooth function on $\mathscr{D}_\Gamma(x)$ with the following Taylor expansion:

$$f_{\Gamma,x}(\xi) = \sum_{s:|s| \leqq k+1} a_{\Gamma,s}(x)\xi^s + r_{\Gamma,k+2}(\xi, x) \tag{13}$$

with the coefficients

$$a_{\Gamma,s}(x) = \frac{d^s f_{\Gamma,x}(0)}{s!\, d\xi^s},$$

where $s = (s_1, \dots, s_{\nu-1})$ is multi-index. Moreover there is a constant $F_\Gamma(k)$, such that uniformaly in $x \in \Gamma$, $|r_{\Gamma,k+2}(\xi, x)| \leqq F_\Gamma(k)\, |\xi|^{k+2}$.

Let

$$I_k(\Lambda) = \sup_{x \in \Gamma} \max \{a_{\Gamma,s}(x) |\Lambda|^{(|s|-1)/\nu}, |s| \leqq k+1; F_\Gamma(k)\, |\Lambda|^{(k+1)/\nu}\}.$$

Note that for any homothetic regions Λ and $\tilde{\Lambda}$, $I_k(\Lambda) = I_k(\tilde{\Lambda})$. Now denote by $\mathscr{B}_a^k$ the class of convex bounded regions $\Lambda \subset \mathbb{R}^\nu$ with a $(k+2)$-smooth boundary such that $I_k(\Lambda) \leqq a$. It is evident that $\delta_\Gamma \geqq \dfrac{|\Lambda|^{1/\nu}}{4a}$ for any $\Lambda \in \mathscr{B}_a^k$.

Let $\mathfrak{M}(\Delta_k)$ be the collection of all non-negative integer-valued finite functions with the supports in Δ_k,

$$\Delta_k = \{s \in \mathbb{Z}_+^{\nu-1} : 2 \leqq |s| \leqq k+1\}, \qquad k \in \mathbb{Z}_+^1,$$

and let

$$\mathfrak{M}_\ell(\Delta_k) = \left\{m \in \mathfrak{M}(\Delta_k) : \sum_s (|s|-1)m(s) = \ell\right\}, \qquad \ell \in \mathbb{Z}_+^1,$$

where $\mathbb{Z}_+^j, j = 1, 2, \dots$ is the semi-group of multi-indexes $s = (s_1, \dots, s_j)$. In addition we put: $(x)_n = (x_1, \dots, x_n) \in (\mathbb{R}^\nu)^n$, $(\xi)_n = (\xi_1, \dots, \xi_n) \in (\mathbb{R}^{\nu-1})^n$, $(\eta)_n = (\eta_1, \dots, \eta_n) \in \mathbb{R}^n$, and $(x)_n = ((\xi)_n, (\eta)_n)$. We also define

$$W_{m,r}((\xi)_n) = \sum_{\substack{m_i \in \mathfrak{M}(\Delta_k) : |m_i| = r_i + 1, \\ m_1 + \dots + m_n = m}} \prod_i \prod_s \frac{\xi_i^{s \cdot m_i(s)}}{m_i!}, \tag{14}$$

where $r=(r_1,\dots,r_n)\in\mathbb{Z}_+^n$, $m\in\mathfrak{M}(\Delta_k)$, $|m|=|r|+n$, $m!=\prod_t m(t)!$. Finally for any cluster function ψ and unit vector $v\in\mathbb{R}^\nu$, we define

$$\mathbb{R}_v^\nu=\{x\in\mathbb{R}^\nu:\langle x,v\rangle>0\};\qquad \omega_v(c)=(H_{\mathbb{R}_v^\nu}\psi)(c),\qquad c\in C_+(\mathbb{R}^\nu).$$

We shall give below the proof (without details, which can be found in [8]) of Theorem 1′ generalizing Theorem 1.

Theorem 1′. *Let* $\Lambda\in\mathscr{B}_a^k$, $1\leqq k\leqq \nu$, $\psi\in\mathscr{K}_{k+1,q,\Lambda_0}^p$, *where*

$$p\geqq \nu+(k+1)^2(k+2)(\nu+2),\qquad q\in\mathscr{N}_3^p$$

and the quantity $\sup_x q(x)$ *is sufficiently small. Then*

$$Q(\Lambda,\psi)=c_0(\psi)\,|\Lambda|+\sum_{\ell=1}^{k}c_\ell(\Lambda,\psi)+R_k(\Lambda,\psi),\tag{15}$$

where $c_0(\psi)$ *is defined by the formula* (11),

$$c_\ell(\Lambda,\psi)=\int_{\Gamma(\Lambda)}\langle n(x),x\rangle b_\ell(x;\Lambda,\psi)d\sigma(x),\qquad \ell=1,\dots,k;$$

$$b_\ell(x;\Lambda,\psi)=\frac{1}{\nu-\ell}\sum_{j=0}^{\ell-1}\sum_{n=1}^{\ell-j}\frac{(-1)^n}{n!}\sum_{m\in\mathfrak{M}_\ell(\Delta_k):|m|=j+n}\prod_s(a_{\Gamma,s}(x))^{m(s)}$$
$$\cdot\sum_{r\in\mathbb{Z}_+^n:|r|=j}\int_{(T_\Gamma(x))^n}(D^r_{(\eta)_n}\omega_{n(x)})(x,(\xi)_n,(0)_n)W_{m,r}((\xi)_n)d(\xi)_n,\quad 1\leqq\ell\leqq\nu-^1;\tag{16}$$

$$b_\nu(x;\Lambda,\psi)=\frac{1}{\nu}\ln|\Lambda|\sum_{j=0}^{\nu-1}\sum_{n=1}^{\nu-j}\frac{(-1)^n}{n!}\sum_{m\in\mathfrak{M}_\nu(\Delta_k):|m|=j+n}\prod_s(a_{\Gamma,s}(x))^{m(s)}$$
$$\cdot\sum_{r\in\mathbb{Z}_+^n:|r|=j}\int_{(T_\Gamma(x))^n}(D^r_{(\eta)_n}\omega_{n(x)})(x,(\xi)_n,(0)_n)W_{m,r}((\xi)_n)d(\xi)_n.$$

Finally

$$|R_k(\Lambda,\psi)|\leqq\begin{cases}B_1(\Lambda_0,q,a)|\Lambda|^{\nu-k-1/\nu}, & k<\nu-1,\\ B_2(\Lambda_0,q,a)\ln|\Lambda|, & k=\nu-1,\\ B_3(\Lambda_0,q,a), & k=\nu.\end{cases}\tag{17}$$

Proof. Without loss of generality we suppose that Λ contains the origin. Consider the family of extended regions $\{\Lambda_L=E_L\Lambda_1, L\geqq 1\}$. It is easy to see that

$$Q(\Lambda_{L+\Delta L},\psi)=\int_{C(K_\Delta L)}\omega_L(c)dc=Q(\Lambda_L,\psi)+\int_{K_{\Delta L}}\omega_L(x)dx+R,$$

where $K_{\Delta L}=\Lambda_{L+\Delta L}\backslash\Lambda_L$, $\Delta L>0$,

$$\omega_L(c)=\int_{C(\Lambda_L)}\psi(c\cup\bar c)d\bar c,$$

$$R=\sum_{n=2}^{\infty}\int_{C^n(K_\Delta L)}\omega_L(c)dc=o(\Delta L),\qquad \Delta L\to 0.$$

Next

$$\int_{K_{\Delta L}}\omega_L(x)dx=\int_L^{L+\Delta L}\frac{dt}{t}\int_{\Gamma_t}\omega_L(x)\langle n(x),x\rangle d\sigma(x)$$
$$=\Delta L\cdot L^{-1}\int_{\Gamma_L}\omega_L(x)\langle n(x),x\rangle d\sigma(x)+o(\Delta L),\qquad \Delta L\to 0.$$

This implies the following formula

$$\frac{d}{dL} Q(\Lambda_L, \psi) = L^{-1} \int_{\Gamma_L} \langle n(x), x \rangle \omega_L(x) d\sigma(x) . \tag{18}$$

Therefore to obtain the desired expansion for $Q(\Lambda, \psi)$, it is sufficient to obtain the asymptotic expansion for the function $\omega_L(x)$.

Let $\tilde{\mathscr{D}}_L(x) = \{\xi \in T_{\Gamma_L}(x) : |\xi| < \delta_{\Gamma_1} L^{1/\nu+2}\}$. Then the functions $f_{L,x} \equiv f_{\Gamma_L,x}$ and f_{1,x_1}, where $x_1 = E_{L^{-1}}(x)$, $x \in \Gamma_L$ are connected by the equality:

$$f_{L,x}(\xi) = L f_{1,x_1}(L^{-1}\xi) , \qquad \xi \in \tilde{\mathscr{D}}_L(x) . \tag{19}$$

By virtue of formula (13) we have

$$f_{L,x}(\xi) = p_{L,x}(\xi) + r_{L,k}(\xi, x) , \qquad \xi \in \tilde{\mathscr{D}}_L(x) ,$$

where the Taylor polynomial for the function $f_{L,x}$,

$$p_{L,x}(\xi) = \sum_{|s| \leqq k+1} a_{L,s}(x) \cdot \xi^s ,$$

and the remainder term satisfies the estimate:

$$|r_{L,k}(\xi, x)| \leqq F_{\Gamma_1}(k) L^{-k-1} |\xi|^{k+2} .$$

Then using the following equality [4]:

$$\int_{C(\mathbb{R}^\nu)} \left(\sum_{c_1 \subset c} \varphi(c_1) \right) f(c) dc = \int_{C(\mathbb{R}^\nu)} \int_{C(\mathbb{R}^\nu)} \varphi(c_1) f(c_1 \cup c_2) dc_1 dc_2 ,$$

we obtain that

$$\omega_L(x) = \int_{C(\mathbb{R}^\nu_{n(x)})} \psi(x \cup c) \prod_{t \in c} (1 - \chi_{\Lambda'_L(x)}(t)) dc = \int_{C(\Lambda'_L(x))} (-1)^{N(c)} \omega_{n(x)}(x \cup c) dc ,$$

where $\Lambda'_L(x) = \mathbb{R}^\nu_{n(x)} \setminus \Lambda_L$. Define

$$\mathscr{P}(x) = \{(\xi, \eta) \in \mathbb{R}^\nu : \xi \in T_{\Gamma_L}(x), 0 < \eta < p_{L,x}(\xi)\} ,$$

and

$$\mathscr{P}_L(x) = \{(\xi, \eta) \in \mathscr{P}(x) : \xi \in \tilde{\mathscr{D}}_L(x)\} .$$

From (20) we obtain that

$$\omega_L(x) = \int_{C(\mathscr{P}_L(x))} (-1)^{N(c)} \omega_{n(x)}(x \cup c) dc + \alpha_1(x, L) ,$$

where by means of Lemma 3 it can be shown that

$$|\alpha_1(x, L)| \leqq B_4(A_0, q, a) L^{-k-1} .$$

By the same Lemma 3 one can show also that

$$\omega_L(x) = \sum_{n=0}^{(k+1)(\nu+2)} (-1)^n \int_{C^n(\mathscr{P}(x))} \omega_{n(x)}(x \cup c) dc + \alpha_2(x, L, k) , \tag{21}$$

where $C^n(\mathscr{P}(x))$ is the set of all n-point configurations in $\mathscr{P}(x)$ and

$$|\alpha_2(x, L, k)| \leqq B_5(A_0, q, a) L^{-k-1} . \tag{22}$$

Next let us consider the integrals

$$J_n(x,L)=\int_{C^n(\mathscr{P}(x))}\omega_{n(x)}(x\cup c)dc$$
$$=\int_{(T_{\Gamma_L}(x))^n}d(\xi)_n\int_0^{p_{L,x}(\xi_1)}d\eta_1\cdot\ldots\cdot\int_0^{p_{L,x}(\xi_n)}\omega_{n(x)}(x,(\xi)_n,(\eta)_n)(\eta)_n^r d\eta_n,$$

where $(\eta)_n^r=\prod_{j=1}^n \eta_j^{r_j}$. Expansion of the function $\omega_{n(x)}(x,(\xi)_n,(\eta)_n)$ in powers of $(\eta)_n$ in a neighbourhood of the point $(x,(\xi)_n,(0)_n)$, and following integration over $(\eta)_n$ implies

$$J_n(x,L)=\frac{1}{n!}\sum_{r\in\mathbb{Z}_+^n:|r|\leqq k}\left(\prod_1^n(r_i+1)!\right)^{-1}$$
$$\cdot\int_{(T_{\Gamma_L}(x))^n}D^r_{(\eta)_n}\omega_{n(x)}(x,(\xi)_n,(0)_n)\prod_i(p_{L,x}(\xi_i))^{r_i+1}d(\xi)_n$$
$$+\frac{1}{n!}\sum_{r\in\mathbb{Z}_+^n:|r|=k+1}\frac{1}{r!}\int_{(T_{\Gamma_L}(x))^n}d(\xi)_n\int_0^{p_{L,x}(\xi_1)}d\eta_1\cdot\ldots$$
$$\cdot\int_0^{p_{L,x}(\xi_n)}D^r_{(\eta)_n}\omega_{n(x)}(x,(\xi)_n,(\theta\eta)_n)(\eta)_n^r d\eta_n,$$

where $(\theta\eta)_n=(\theta\eta_1,\ldots,\theta\eta_n)$, $0<\theta<1$. Denote the first and second sums in (23) by $J_n'(x,L)$ and $J_n''(x,L)$, respectively. Using the following evident identity:

$$\prod_i(p_{i,x}(\xi_i))^{r_i+1}=\prod_i(r_i+1)!\sum_{\ell=|r|+n}^{k(|r|+n)}L^{-\ell}\sum_{\substack{m\in\mathfrak{M}_\ell(\Delta_k):\\|m|=|r|+n}}\left(\prod_s(a_{1,s}(x))^{m(s)}\right)W_{m,r}((\xi)_n),$$

where the quantities $W_{m,r}((\xi)_n)$ are defined by formula (14), we get

$$\sum_{n=1}^{(k+1)(v+2)}(-1)^nJ_n'(x,L,k)=\sum_{\ell=1}^k\tilde{c}_\ell(x,\Lambda_L,\psi)+\alpha_3(x,L,k). \tag{24}$$

Here

$$|\alpha_3(x,L,k)|\leqq B_6(A_{0,q,a})L^{-k-1}, \tag{25}$$

and the quantities $\tilde{c}_\ell$ are equal to:

$$\begin{gathered}\tilde{c}_\ell(x,\Lambda_L,\psi)=(v-\ell)b_\ell(x,\Lambda_L,\psi), \quad \ell=1,\ldots,v-1;\\ \tilde{c}_v(x,\Lambda_L,\psi)=vb_v(x,\Lambda_L,\psi)\ln^{-1}|\Lambda|;\end{gathered} \tag{26}$$

where b_ℓ are defined by formula (16). Note that the following relations are valid:

$$\tilde{c}_\ell(x,\Lambda_L,\psi)=L^{-\ell}\tilde{c}_\ell(x_1,\Lambda_1,\psi), \quad \ell=1,\ldots,v,$$

with $x_1=E_{L^{-1}}(x)$.

On the other hand

$$\left|\sum_{n=1}^{(k+1)(v+2)}(-1)^nJ_n''(x,L,k)\right|\leqq B''(A_0,q,a)L^{-k-1}. \tag{27}$$

Thus from (21)–(27) we obtain the asymptotic expansion for the function $\omega_L(x)$:

$$\omega_L(x)=\sum_{\ell=0}^{k}\tilde{c}_\ell(x,\Lambda_L,\psi)+\alpha_4(x,L,k)\,, \tag{28}$$

where

$$|\alpha_4(x,L,k)|\leqq B_7(\Lambda_0,q,a)L^{-k-1}\,, \tag{29}$$

and the quantity $\tilde{c}_0$ has the form

$$\tilde{c}_0(x,\Lambda_L,\psi)\equiv\tilde{c}_0(x,\psi)=\int_{C(\mathbb{R}^\nu_{n(x)})}\psi(x\cup c)dc\,. \tag{30}$$

Substituting (28) in (18) one finds that

$$\begin{aligned}Q(\Lambda,\psi)&=\int_1^{|\Lambda|^{1/\nu}}\frac{d}{dL}Q(\Lambda_L,\psi)dL+Q(\Lambda_1,\psi)=\int_{\Gamma(\Lambda)}\tilde{c}_0(x,\psi)\langle n(x),x\rangle d\sigma(x)\\&\quad+\sum_{\ell=1}^{k}c_\ell(\Lambda,\psi)+R_k(\Lambda,\Psi)\,,\end{aligned} \tag{31}$$

where

$$\begin{aligned}R_k(\Lambda,\psi)&=Q(\Lambda_1,\psi)-\sum_{\ell=1}^{k}c_\ell(\Lambda_1,\psi)-\int_{\Gamma_1}\tilde{c}_0(x,\psi)\langle n(x),x\rangle d\sigma(x)\\&\quad+\int_1^{|\Lambda|^{1/\nu}}L^{-1}dL\int_{\Gamma_L}\alpha_4(x,L,k)\langle n(x),x\rangle d\sigma(x)\,.\end{aligned}$$

The estimate (17) for the quantity $R_k(\Lambda,\psi)$ can be obtained with the help of Lemma 3 and the well-known Gauss-Ostrogradsky's formula. Finally note that the comparison of Theorem 7 with expansion (31) implies the equality

$$\int_{\Gamma(\Lambda)}\tilde{c}_0(x,\psi)\langle n(x),x\rangle d\sigma(x)=c_0(\psi)\,.$$

Thus Theorem 1′ is proved.

Proof of Theorem 2. Evidently from the spherical symmetry of the potential Φ it follows that the corresponding Ursell function is Euclidean invariant. Hence for any $v\in\mathbb{R}^{\nu-1}$ the function ω_v is also Euclidean invariant. Therefore by virtue of (26) and (31),

$$\begin{aligned}c_1(\Lambda,\psi)&=-\frac{1}{\nu-1}\sum_{s\in\mathbb{Z}_+^{\nu-1}:|s|=2}\frac{1}{s!}\int_{T_\Gamma(x)}\omega_{n(x)}(x,\xi,0)\xi^s d\xi\\&\quad\cdot\int_{\Gamma(\Lambda)}\frac{d^s f_{\Gamma,x}}{d\xi^s}(0)\langle n(x),x\rangle d\sigma(x)\,.\end{aligned}$$

Now put

$$d_1(\psi)=-\tfrac{1}{2}\int_{\mathbb{R}^{\nu-1}}\langle\xi,e\rangle^2\omega_v(0,\xi)d\xi\,, \tag{32}$$

where e and v are arbitrary unit vectors in $\mathbb{R}^{\nu-1}$. Then

$$c_1(\Lambda,\psi)=\frac{d_1(\psi)}{\nu-1}\int_{\Gamma(\Lambda)}\langle n(x),x\rangle\sum_{i=1}^{\nu-1}(\kappa_\Gamma)_i(x)d\sigma(x)=d_1(\psi)S(\Gamma(\Lambda)).$$

Here we use Minkovsky's formula for the area of the surface of convex region with smooth boundary (see [5]).

Next for the case $\nu>2$ we have

$$\begin{aligned}c_2(\Lambda,\psi)=\frac{1}{\nu-2}\Bigg\{&-\sum_{s:|s|=3}\frac{1}{s!}\int_{\Gamma(\Lambda)}\langle n(x),x\rangle\frac{d^s f_{\Gamma,x}}{d\xi^s}(0)d\sigma(x)\int_{T_\Gamma(x)}\omega_{n(x)}(x,\xi,0)\xi^s d\xi\\&+\frac{1}{2}\sum_{s_1,s_2:|s_1|=|s_2|=2}\frac{1}{s_1!s_2!}\int_{\Gamma(\Lambda)}\langle n(x),x\rangle\frac{d^{s_1}f_{\Gamma,x}}{d\xi_1^{s_1}}(0)\frac{d^{s_2}f_{\Gamma,x}}{d\xi_2^{s_2}}(0)d\sigma(x)\\&\cdot\int_{T_\Gamma(x)}\int_{T_\Gamma(x)}\omega_{n(x)}(x,\xi_1,\xi_2,0,0)\xi_1^2\xi_2^2 d\xi_1 d\xi_2-\frac{1}{2}\sum_{s_1,s_2:|s_1|=|s_2|=2}\frac{1}{s_1!s_2!}\\&\cdot\int_{\Gamma(\Lambda)}\langle n(x),x\rangle\frac{d^{s_1}f_{\Gamma,x}}{d\xi_1^{s_1}}(0)\frac{d^{s_2}f_{\Gamma,x}}{d\xi_2^{s_2}}(0)d\sigma(x)\int_{T_\Gamma(x)}D_\eta\omega_{n(x)}(x,\xi,0)\xi^{(s_1+s_2)}d\xi\Bigg\}.\end{aligned}$$

On the other hand, since

$$\int_{T_\Gamma(x)}\omega_{n(x)}(x,\xi,0)\xi^s d\xi=0,\quad\text{for}\quad |s|=3\quad\text{and}\quad D_\eta\omega_{n(x)}(x,\xi,0)=0.$$

Then

$$c_2(\Lambda,\psi)=\frac{1}{\nu-2}[d_2(\psi)M(\Gamma(\Lambda))+\tilde{d}_2(\psi)\tilde{M}(\Gamma(\Lambda))],$$

where the quantity M is defined by formula (7) while

$$\begin{aligned}\tilde{M}(\Gamma(\Lambda))&=\int_{\Gamma(\Lambda)}\langle n(x),x\rangle\sum_{i=1}^{\nu-1}(\kappa_\Gamma)_i^2(x)d\sigma(x),\\ d_2(\psi)&=\tfrac{1}{4}\int_{\mathbb{R}^{\nu-1}}\int_{\mathbb{R}^{\nu-1}}\langle\xi_1 e_1\rangle^2\langle\xi_2,e_2\rangle^2\omega_v(0,\xi_1,\xi_2)d\xi_1 d\xi_2,\\ \tilde{d}_2(\psi)&=\tfrac{1}{8}\int_{\mathbb{R}^{\nu-1}}\int_{\mathbb{R}^{\nu-1}}\langle\xi_1,e\rangle^2\langle\xi_2,e\rangle^2\omega_v(0,\xi_1,\xi_2)d\xi_1 d\xi_2,\end{aligned}\tag{33}$$

where $e_1,e_2\in\mathbb{R}^{\nu-1}$ is an arbitrary pair of orthonormal vectors. (It is evident that d_2 and $\tilde{d}_2$ do not depend on the choice of this pair.) From the translation invariance of the potential Φ it follows that $c_2(\Lambda,\psi)$ is also translation invariant. Combining this with the easily checked fact that in general the quantity $\tilde{M}$ depends on the choice of the origin, we obtain that $\tilde{d}_2(\psi)=0$.

In a similar way one can consider the case $\nu=2$. So Theorem 2 is proved.

Remark. For simplicity we will prove following Theorems 3–5 for the case $\nu=2$.

Proof of Theorem 3. Let $\Lambda\subset\mathbb{R}^2$ be a convex region with piecewise smooth boundary. In every curvilinear angle corresponding to the point of discontinuity of $\Gamma(\Lambda)$ we inscribe an arc of a circle of radius $r|\Lambda|^{1/6}$, where $r>0$ is a constant. The "smoothed" region obtained in this way is denoted by $\tilde{\Lambda}$.

Lemma 4. *For any Euclidean invariant* $\psi \in \mathscr{K}^p_{2,q,\Lambda_0}$, $p \geqq 50$, *where the upper bound of the function* $q \in \mathscr{N}^p_3$ *is sufficiently small, we have:*

$$Q(\hat{\Lambda}_L, \psi) = c_0(\psi)\,|\hat{\Lambda}_L| + d_1(\psi)S(\Gamma(\hat{\Lambda}_L)) + R_1(\hat{\Lambda}_L, \psi)\,, \tag{34}$$

where the coefficients c_0 *and* d_1 *are defined by formulae* (11) *and* (32) *respectively. Moreover*

$$|R_1(\hat{\Lambda}_L, \psi)| \leqq B(\Lambda)L^{11/12}\,. \tag{35}$$

Proof. Without the loss of generality one can suppose that $\Gamma(\Lambda)$ has only one point of discontinuity. Let x_1 and x_2 be the points of tangency of the inscribed arc with $\Gamma(\Lambda)$. Define:

$$\hat{\Gamma}'_L = \{x \in \hat{\Gamma}_L : \varrho(x, x_1) < \delta L^{1/4}\} \cup \{x \in \hat{\Gamma}_L : \varrho(x, x_2) < \delta L^{1/4}\}\,,$$

$\hat{\Gamma}''_L = \hat{\Gamma}_L \backslash \hat{\Gamma}'_L$, $L \geqq 1$, where $\delta = \delta_{\hat{\Gamma}_1}$, $\hat{\Gamma}_L = \Gamma(\hat{\Lambda}_L)$. By repeating the arguments used in the proof of Theorem 1 one can show that

$$\hat{\omega}_L(x) = \hat{c}_0(x) + \hat{c}_1(x)\kappa_{\hat{\Gamma}_L}(x) + \hat{R}_1(x, L)\,, \qquad x \in \hat{\Gamma}_L\,, \tag{36}$$

where $\hat{c}_0(x) = \tilde{c}_0(x)$ [see formula (30)], $\hat{\omega}_L \equiv \omega_{\hat{\Lambda}_L}$,

$$\hat{c}_1(x) = -\tfrac{1}{2} \int\limits_{T\hat{\Gamma}_L(x)} \omega_{n(x)}(x, \xi, 0)\xi^2 d\xi\,,$$

and

$$|\hat{R}_1(x, L)| \leqq \begin{cases} \hat{B}_1(\Lambda_0, q, a)\kappa^2_{\hat{\Gamma}_L}(x)\,, & x \in \hat{\Gamma}''_L\,, \\ \hat{B}_2(\Lambda_{0,q,a})L^{-1/3}\,, & x \in \hat{\Gamma}'_L\,. \end{cases} \tag{37}$$

The formulae (36) and (18) imply expansion (34). To obtain the estimate (35) note that by virtue of (37) and (18)

$$\begin{aligned} \int\limits_{\hat{\Gamma}_L} |\hat{R}_1(x, L)|\,\langle n(x), x\rangle d\sigma(x) &\leqq \hat{B}_1(\Lambda_0, q, a)L^{-1/3} \int\limits_{\hat{\Gamma}''_L} \langle n(x), x\rangle \kappa_{\hat{\Gamma}_L}(x) d\sigma(x) \\ &+ \hat{B}_2(\Lambda_0, q, a)L^{-1/3} \int\limits_{\hat{\Gamma}'_L} \langle n(x), x\rangle d\sigma(x) \leqq \hat{B}_3(\Lambda_0, q, a, \Lambda)L^{11/12}\,. \end{aligned}$$

On the other hand

$$\begin{aligned} R_1(\hat{\Lambda}_L, \psi) &= Q(\hat{\Lambda}_1, \psi) - c_0(\psi) - d_1(\psi)S(\Gamma(\hat{\Lambda}_1)) \\ &+ \int\limits_1^{|\hat{\Lambda}_L|^{1/2}} L^{-1} dL \int\limits_{\hat{\Gamma}_L} \hat{R}_1(x, L)\,\langle n(x), x\rangle d\sigma(x)\,. \end{aligned}$$

So Lemma 4 is proved.

Now it is easy to see that the following estimates are valid:

$$|\Lambda_L \backslash \hat{\Lambda}_L| \leqq B_1(\Lambda)L^{2/3}\,, \qquad S(\Gamma(\Lambda_L) \backslash \Gamma(\hat{\Lambda}_L)) \leqq B_2(\Lambda)L^{1/3}\,. \tag{38}$$

Moreover

$$\begin{aligned} |Q(\Lambda_L, \psi) - Q(\hat{\Lambda}_L, \psi)| &\leqq \int\limits_{C(\Lambda_L \backslash \hat{\Lambda}_L)} dc_1 \int\limits_{C_+(\hat{\Lambda}_L)} \psi(c_1 \cup c_2) dc_2 \\ &\leqq \int\limits_{\Lambda_L \backslash \hat{\Lambda}_L} dx \int\limits_{C(\Lambda_L \backslash \hat{\Lambda}_L)} \frac{(H_{\hat{\Lambda}_L}|\psi|)(x \cup c)}{N(c) + 1} dc \leqq B_3(\Lambda_0, q)|\Lambda_L \backslash \hat{\Lambda}_L|\,. \end{aligned}$$

Together with (38) this implies the estimate:

$$|Q(\hat{\Lambda}_L, \psi) - Q(\Lambda_L, \psi)| \leqq B_4(A_0, q, \Lambda)L^{2/3}. \tag{39}$$

Combining (38) and (39) we obtain Theorem 3.

Proof of Theorem 4. Consider a convex polyhedron $\Lambda \subset \mathbb{R}^2$ of the class $\mathscr{C}(r, r', n_0)$. Let $M_1, \ldots, M_n$ be the vertexes and $\theta_1, \ldots, \theta_n$ be the corresponding angles of Λ. Denote by Π_i the half-plane which is defined by the straight line M_iM_{i+1}, and which does not contain Λ $(M_{n+1} = M_1)$.

Let us put: $G_i = \mathbb{R}^2 \backslash (\Pi_i \cup \Pi_{i+1})$, $\tilde{G}_i = \Pi_i \cap \Pi_{i+1}$, $i = 1, \ldots, n$; $\Pi_{n+1} = \Pi_1$.

For any Euclidean invariant $\psi \in \mathscr{K}^p_{q,A}$, $p \geqq 50$, if the upper bound of the function $q \in \mathscr{N}^p_3$ is sufficiently small, we have:

$$Q(\Lambda, \psi) = \int_\Lambda dx \int_{C(\Lambda')} \tilde{\psi}(x \cup c)dc, \tag{40}$$

where $\Lambda' = \mathbb{R}^2 \backslash \Lambda$,

$$\tilde{\psi}(c) = (-1)^{N(c)+1} \int_{C(\mathbb{R}^2)} \frac{\psi(c \cup \bar{c})}{N(c) + N(\bar{c})} d\bar{c}, \qquad c \in C_+(\mathbb{R}^2).$$

Then

$$\begin{aligned}\int_{C(\Lambda')} \tilde{\psi}(x \cup c)dc = c_0(\psi) + \sum_{i=1}^n \int_{C_+(\Pi_i)} \tilde{\psi}(x \cup c)dc - \sum_{i=1}^n \int_{C_+(\tilde{G}_i)} \tilde{\psi}(x \cup c)dc \\ + \sum_{i=1}^n \int_{\tilde{C}_+(\Pi_i \cup \Pi_{i+1})} \tilde{\psi}(x \cup c)dc + R(\Lambda, \psi),\end{aligned} \tag{41}$$

where $c_0(\psi)$ is defined by formula (11),

$$\tilde{C}_+(\Pi_i \cup \Pi_{i+1}) = C_+(\Pi_i \cup \Pi_{i+1}) \backslash (C_+(\Pi_i) \cup C_+(\Pi_{i+1})), \qquad i = 1, \ldots, n.$$

Let us estimate $R(\Lambda, \psi)$:

$$|R(\Lambda, \psi)| \leqq \int_{c \in C(\Lambda') : \operatorname{diam} c \geqq \sigma(\Lambda)} |\tilde{\psi}(x \cup c)| dc \leqq \int_{c \in C(\mathbb{R}^2) : c \cap S'_x \neq \emptyset} |\tilde{\psi}(x \cup c)| dc.$$

Here $S'_x = \mathbb{R}^2 \backslash S_x$, where S_x is the ball of radius $\frac{\sigma(\Lambda)}{2}$ about x. By Lemma 2 we obtain

$$|R(\Lambda, \psi)| \leqq C(q, r)|\Lambda|^{(2-p)/2} \tag{42}$$

Now substituting (41) in (40) we get:

$$\begin{aligned}Q(\Lambda, \psi) = c_0(\psi)\,|\Lambda| + \sum_{i=1}^n \int_\Lambda dx \int_{C_+(\Pi_i)} \tilde{\psi}(x \cup c)dc - \sum_{i=1}^n \int_\Lambda dx \int_{C_+(\tilde{G}_i)} \tilde{\psi}(x \cup c)dc \\ + \sum_{i=1}^n \int_\Lambda dx \int_{\tilde{C}_+(\Pi_i \cup \Pi_{i+1})} \tilde{\psi}(x \cup c)dc + R_1(\Lambda, \psi),\end{aligned} \tag{43}$$

where

$$|R_1(\Lambda, \psi)| \leqq C(q, r)\,|\Lambda|^{(4-p)/2}.$$

Let us investigate the first sum of (43). Let $e, v \in \mathbb{R}^2$ be an arbitrary pair of orthonormal vectors and (ξ, η) be the corresponding system. For any

$\theta, -\frac{\pi}{2}<\theta<\frac{\pi}{2}$ consider the ray drawn from the origin which makes with η axes angle θ. Let us denote the obtained angular region by Y_θ. Now put

$$\bar{c}_1(\psi)=\int_0^\infty d\eta \int_{C_+(\mathbb{R}^2_v)} \tilde{\psi}((\xi,\eta)\cup c)dc\,.$$

It is evident that this integral converges and does not depend on the choice of ξ and v. Furthermore

$$\begin{aligned}\int_\Lambda dx \int_{C_+(\Pi_i)} \tilde{\psi}(x\cup c)dc &= \bar{c}_1(\psi)\,|M_iM_{i+1}| - \operatorname{sgn}(\cos\theta_i) \int_{Y_{\theta_i-\frac{\pi}{2}}} dx \int_{C_+(\mathbb{R}^2_v)} \tilde{\psi}(x\cup c)dc \\ &\quad - \operatorname{sgn}(\cos\theta_{i+1}) \int_{Y_{\frac{\pi}{2}-\theta_{i+1}}} dx \int_{C_+(\mathbb{R}^2_v)} \tilde{\psi}(x\cup c)dc + R_i'(\Lambda,\psi)\,,\end{aligned} \tag{44}$$

$i=1,\dots,n$, where $|M_iM_{i+1}|$ denotes the length of the side M_iM_{i+1} and it is easy to check that the quantities R_i' satisfy the following estimates

$$|R_i'(\Lambda,\psi)| \leqq C_2|\Lambda|^{(4-p)/2}\,,$$

with the constant $C_2=C_2(q,r,r')$.

Similarly for the second and the third summands of (43) we obtain that for

$$\begin{aligned}\int_\Lambda dx \int_{C_+(\tilde{G}_i)} \tilde{\psi}(x\cup c)dc &= \int_{G_i} dx \int_{C_+(\tilde{G}_i)} \tilde{\psi}(x\cup c)dc + R_i''(\Lambda,\psi)\,, \\ \int_\Lambda dx \int_{C_+(\Pi_i\cup\Pi_{i+1})} \tilde{\psi}(x\cup c)dc &= \int_{G_i} dx \int_{C_+(\Pi_i\cup\Pi_{i+1})} \tilde{\psi}(x\cup c)dc + R_i'''(\Lambda,\psi)\,,\end{aligned} \tag{45}$$

where

$$|R_i''(\Lambda,\psi)| \leqq C_i''(q,r,r')\,|\Lambda|^{(4-p)/2}\,,$$

and

$$|R_i'''(\Lambda,\psi)| \leqq C_i'''(q,r,r')\,|\Lambda|^{(4-p)/2}\,.$$

Hence combining (43)–(45) we get:

$$Q(\Lambda,\psi)=c_0(\psi)\,|\Lambda| + c_1(\psi)S(\Gamma(\Lambda)) + c_2(\theta_1,\dots,\theta_n,\psi) + R(\Lambda,\psi)\,.$$

Here

$$\begin{aligned}C_2(\theta_1,\dots,\theta_n,\psi) &= -\sum_{i=1}^n \operatorname{sgn}(\cos\theta_i) \int_{Y_{\theta_i}\cup Y_{-\theta_i}} dx \int_{C_+(\mathbb{R}^2_v)} \tilde{\psi}(x\cup c)dc \\ &\quad + \sum_{i=1}^n \int_{G_i} dx \int_{C_+(\tilde{G}_i)} \tilde{\psi}(x\cup c)dc + \sum_{i=1}^n \int_{G_i} dx \int_{C_+(\Pi_i\cup\Pi_{i+1})} \tilde{\psi}(x\cup c)dc\,.\end{aligned}$$

In addition

$$|R(\Lambda,\psi)| \leqq C(q,r,r',n_0)\,|\Lambda|^{-1/2}\,.$$

Thus Theorem 4 is proved.

Now consider the lattice case. For this case we have the following analogy of the formula (12):

$$\ln \Xi(\Lambda,\beta,z) = \sum_{X\in\mathfrak{M}(\mathbb{Z}^\nu)} \frac{\psi_{\beta,z}(X)}{X!}\,, \tag{46}$$

where $\psi_{\beta,z}$ is the corresponding Ursell function. We recall that $\mathfrak{M}(\mathbb{Z}^\nu)$ is the collection of all non-negative integer-valued finite functions on $\mathbb{Z}^\nu$. So we investigate the asymptotic behaviour of the quantity

$$Q(\Lambda,\psi)=\sum_{X\in\mathfrak{M}(\mathbb{Z}^\nu)}\frac{\psi(X)}{X!}, \tag{47}$$

where ψ is a cluster function and $X!=\prod_{t\in\mathbb{Z}^\nu}X(t)!$.

Proof of Theorem 5. Suppose that the cluster function $\psi\in\mathcal{K}^p_{q,\Lambda}$, $p\geqq 5$, where the upper bound of the function $q\in\mathcal{N}^p_3$ is sufficiently small. For any function $X\in\mathfrak{M}(\mathbb{Z}^2)$ we denote the support of X by $\tilde{X}$. Then

$$Q(\Lambda,\psi)=\sum_{c\subset\Lambda}\varphi(c),$$

where

$$\varphi(c)=\sum_{X\in\mathfrak{M}(\mathbb{Z}^2):\tilde{X}=c}\frac{\psi(X)}{X!}.$$

Hence

$$Q(\Lambda,\psi)=\sum_{x\in\Lambda}\sum_{c\subset\Lambda\setminus x}\frac{\varphi(x\cup c)}{N(c)+1}.$$

On the other hand

$$\sum_{c\subset\Lambda\setminus x}\frac{\varphi(x\cup c)}{N(c)+1}=\sum_{c\in C(\Lambda')}\varphi_1(x\cup c),$$

where

$$\varphi_1(x\cup c)=(-1)^{N(c)}\sum_{\bar{c}\in C(\mathbb{Z}^2\setminus\{x\cup c\})}\frac{\varphi(x\cup c\cup\bar{c})}{N(c)+N(\bar{c})+1}.$$

It is easy to see that

$$\varphi_1(x\cup c)=(-1)^{N(c)}\sum_{-c\in C(\mathbb{Z}^2\setminus\{x\cup c\})}\frac{1}{N(c)+N(\bar{c})+1}\sum_{\substack{X_1,X_2\in\mathfrak{M}(\mathbb{Z}^2):\\ \tilde{X}_1=c\cup x,\,\tilde{X}_2=c}}\frac{\psi(X_1+X_2)}{X_1!X_2!}$$

$$=(-1)^{N(c)}\sum_{X\in\mathfrak{M}(\mathbb{Z}^2):\tilde{X}=c\cup x}\frac{\varphi_2(X)}{X!},$$

where

$$\varphi_2(X)=\sum_{Y\in\mathfrak{M}(\mathbb{Z}^2\setminus\tilde{X})}\frac{1}{N(\tilde{X})+N(\tilde{Y})}\frac{\psi(X+Y)}{Y!}.$$

Thus

$$Q(\Lambda,\psi)=\sum_{x\in\Lambda}\sum_{c\in C(\Lambda')}f(x\cup c), \tag{48}$$

where

$$f(x\cup c)=(-1)^{N(c)}\sum_{X\in\mathfrak{M}(\mathbb{Z}^2):\tilde{X}=c\cup x}\frac{\varphi_2(X)}{X!}.$$

Now from the strong cluster estimate for truncated correlation functions obtained in [7] it follows that $f(c)$ is also a cluster function. From this and formula (48) by repeating the previous arguments in the proof of Theorem 4 one can obtain the following asymptotic expansion:

$$Q(\Lambda,\psi)=c_0(\psi)\,|\Lambda|+c_1(\psi)S(\Gamma(\Lambda))+c_2(\psi)+R(\Lambda),$$

where

$$c_0(\psi)=\sum_{X\in\mathfrak{M}(\mathbb{Z}^2):\tilde{X}=c\cup x}\frac{\psi(X)}{(N(c)+1)X!},$$

$$c_1(\psi)=\sum_{x\in(\mathbb{Z}^1)_+}\sum_{c\in C(\mathbb{Z}^2_\perp)}f(x\cup c),$$

$$c_2(\psi)=\sum_{x\in\bar{\mathbb{Z}}^2_{+,+}}\left(\sum_{c\in C(\mathbb{Z}^2_{-,-})}f(x\cup c)+\sum_{c\in C_+(G)}f(x\cup c)\right).$$

Here

$$(\mathbb{Z}^1)_+=\{(x^1,x^2)\in\mathbb{Z}^2:x^1=0,x^2\geqq 0\},\quad \mathbb{Z}^2_\perp=\{(x^1,x^2)\in\mathbb{Z}^2:x^2<0\},$$

$$\mathbb{Z}^2_{+,-}=\{(x^1,x^2)\in\mathbb{Z}^2:x^1>0,x^2<0\},\quad \bar{\mathbb{Z}}^2_{+,+}=\{x^1,x^2)\in\mathbb{Z}^2:x^1\geqq 0,x^2\geqq 0\},$$

$$G=\{c\in C(\mathbb{Z}^2\backslash\bar{\mathbb{Z}}^2_{+,+}):c\cap\mathbb{Z}^2_{-,-}\neq\emptyset,\quad c\cap\mathbb{Z}^2_{+,-}\neq\emptyset\},$$

while the notations $\mathbb{Z}^2_{-,+}$, $\mathbb{Z}^2_{+,+}$, and $\mathbb{Z}^2_{-,-}$ have an analogous sense. So Theorem 5 is proved.

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References

1. Yang, C.N., Lee, T.D.: Statistical theory of equations of state and phase transitions. I. Theory of condensation. Phys. Rev. **87**, 404–409 (1952)
2. Dobrushin, R.L.: Asymptotical behaviour of gibbsian distributions for lattice systems and its dependence on the form of the volume. Teor. Mat. Fiz. **1**, 115–134 (1972) (in Russian)
3. Ruelle, D.: Statistical mechanics. Rigorous results. New York, Amsterdam: W. A. Benjamin Inc. 1969
4. Minlos, R.A.: Lectures on statistical physics. Usp. Mat. Nauk. **23**, 137–194 (1968) (in Russian)
5. Santalo, L.: Integral geometry and geometric probability. Reading, MA: Addison-Wesley 1976
6. Duneau, M., Souillard, B.: Cluster properties of lattice and continuous systems. Commun. Math. Phys. **47**, 155–166 (1976)
7. Duneau, M., Iagolnitzer, D., Souillard, B.: Decay of correlations for infinite-range interactions. J. Math. Phys. **16**, 1662–1666 (1975)
8. Minlos, R.A., Pogosian, S.K.: Estimates of Ursell functions, group functions, and their derivatives. Teor. Mat. Fiz. **31**, 199–213 (1977) (in Russian)
9. Pogosian, S.K.: Asymptotic expansion of the logarithm of partition function. Izv. AN Arm. SSR (Ser. Math.) **13**, 238–263 (1978) (in Russian)

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