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Erratum

Kossakowski, A., Frigerio, A., Gorini, V., Verri, M.: Quantum Detailed Balance and KMS Condition. Commun. math. Phys. **57**, 97—110 (1977)

In the proof of Theorem 2.2, Equation (2.14) is wrong. To recover the correct proof, replace the part from line 1 after formula (2.13) ("Let $H_N = \ldots$") to line -8 from the end of the proof ("...boundedness estimates") by the following:

Let

$$C_{rr'ss'} = \tfrac{1}{2}(K_{rr'ss'} + K_{s'sr'r}\varrho_{s'}\varrho_s^{-1}) = K_{rr'ss'} + i(\varepsilon_r - \varepsilon_s)\delta_{rr'}\delta_{ss'}\ ,$$

the equality of the two expressions following from Equation (2.13). By construction, $\{C_{rr'ss'}\}$ satisfies (2.12), and moreover

$$\sum_{rr'ss'=1}^{N} C_{rr'ss'}P_{rr'}AP_{s's} = E_N\Psi E_N(A) + i[E_N(H), E_N(A)]\ ,$$

where $E_N(A) = Q_N A Q_N$, $Q_N = \sum_{r=1}^{N} P_{rr}$.

Notice that $E_N(A)$ converges ultraweakly to A as $N \to \infty$.

In particular, setting $A = \mathbb{1}$,

$$\sum_{rr'ss'=1}^{N} C_{rr'ss'}P_{rr'}P_{s's} = E_N\Psi(Q_N)\ ,$$

which converges ultraweakly as $N \to \infty$ to $\Psi(\mathbb{1})$, thus proving (2.11). Indeed,

$$\lim_{N\to\infty} \mathrm{Tr}\{A_0[E_N\Psi(Q_N) - \Psi(\mathbb{1})]\} = 0$$

for all A_0 in the linear span of the P_{rs}'s, which is dense in the space of trace class operators on $\mathscr{H}$, and $|\mathrm{Tr}\{A_0[E_N\Psi(Q_N) - \Psi(\mathbb{1})]\}|$ is bounded by $2\|A_0\|_1\,\|\Psi\|$.

Using the same kind of arguments, one proves that the expression

$$\sum_{rr'ss'=1}^{N} C_{rr'ss'}(P_{rr'}AP_{s's} - \tfrac{1}{2}\{P_{rr'}P_{s's}, A\})$$
$$= E_N\Psi E_N(A) - \tfrac{1}{2}\{E_N\Psi(Q_N), A\} + iE_N([H, E_N(A)])$$

tends to $L_s(A)$ ultraweakly as $N \to \infty$.

Finally, for any finite sequence $\{x_{rr'}\}$ we have

$$\sum_{rr'ss'=1}^{N} x_{rr'}C_{rr'ss'}x_{ss'}$$
$$= \tfrac{1}{2}\sum_j \left\{ \left| \sum_{rr'} x_{rr'}\langle r'|V_j|r\rangle \right|^2 + \left| \sum_{rr'} y_{rr'}\overline{\langle r|V_j|r'\rangle} \right|^2 \right\} \geqq 0\,,$$

where $y_{rr'} = x_{rr'}(\varrho_{r'}\varrho_r^{-1})^{1/2}$ (we have used the fact that $K_{rr'ss'} = 0$ if $\varrho_{r'}\varrho_r^{-1} \neq \varrho_{s'}\varrho_s^{-1}$). The inequality clearly holds also for those infinite sequences for which the expression converges. This proves (2.10).