never an operator norm. Moreover, a unital ring norm need not be the supremum of the operator norms it majorizes. There is an interesting characterization of those that are, as well as of the ring norms that majorize a unique operator norm. Maximal chains of ring norms have the order type of $[0, +\infty)$ and, finally, any order automorphism of the set of all ring norms is inner, in the sense that it is induced by an automorphism or antiautomorphism of the matrix algebra. The proof of the latter, which is lengthy, involves extending the automorphism to seminorms which are allowed to take the value $+\infty$. Thus a map of subalgebras, the domains of finiteness of these seminorms, is induced, from which it is possible eventually to construct the desired algebra automorphism or antiautomorphism.

REFERENCES

1. I. M. Gelfand, *Normierte ringe*, Mat. Sb. **9** (1941), 3–24.
2. I. M. Gelfand and M. A. Naimark, *On the embedding of normed rings into the ring of operators in Hilbert space*, Mat. Sb. **12** (1943), 197–213.
3. J. Marik and V. Ptak, *Norms, spectra, and combinatorial properties of matrices*, Czechoslovak Math. J. **10** (1960), 181–196.
4. F. J. Murray and J. von Neumann, *On rings of operators*, Ann. of Math. **37** (1936), 116–229.
5. J. von Neumann, *Some matrix-inequalities and metrization of matric-space*, Tomsk Univ. Rev. **1** (1937), 286–300; also *Collected works*, vol. IV, Pergamon (1962), 205–219.
6. R. Schatten, *A theory of cross-spaces*, Ann. of Math. Stud., no. 26, Princeton Univ. Press, Princeton, N.J., 1950.
7. ——, *Norm ideals of completely continuous operators*, Ergeb. Math. Grenzgeb., Springer-Verlag, New York, 1960.

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Vibration and coupling of continuous systems: asymptotic methods, by Jacqueline Sanchez Hubert and Enrique Sanchez Palencia. Springer-Verlag, Berlin, New York, 1989, 420 pp., $79.00. ISBN 3-450-19384-7

We have all been exposed at one time or another to a study of the motion of n point masses connected by linear springs and dampers in three-dimensional physical space. The system of differential equations governing such motion is

$$\text{(E)} \qquad M\ddot{y}(t) + D\dot{y}(t) + Ky(t) = f(t),$$

where M, D, and K are $3n \times 3n$ matrices representing the mass, dissipative friction coefficients, and stiffness of the springs. Here y is a $3n$-vector denoting the displacement of each mass point in $\mathbf{R}^3$ at time t from a fixed reference configuration. The $3n$-vector f is the applied body force, e.g. gravity. Existence and uniqueness of solutions of (E) are readily proven via the theory of ordinary differential equations under reasonable assumptions on M, D, K, and f. Stability of the zero equilibrium when $f \equiv 0$ is determined by the behavior of the ansatz solution $y(t) = e^{\lambda t} v$, $v \in \mathbf{R}^{3n}$, i.e. the value λ satisfying

$$\text{(S)} \qquad (M\lambda^2 + D\lambda + K)v = 0$$

for some nontrivial eigenvector v.

If we imagine an elastic "continuous" body as a limit of an infinite number of such masses, dampers, and springs, it is natural to expect the displacement $u(x, t) \in \mathbf{R}^3$, $x \in \mathbf{R}^3$, $t \in \mathbf{R}$, to satisfy the limiting form of (E), i.e. an evolution equation

$$\text{(L)} \qquad M\ddot{u} + D\dot{u} + Ku = f,$$

where M, D, and K are linear operators (D and K are often unbounded) on a suitable infinite-dimensional vector space H (usually chosen to be a Hilbert space). Now u and f are elements of H for each value of t. Elegant examples of this line of reasoning are found in the book of Gallovotti [1].

Boundary conditions associated with a continuous body are subsumed by the domains of definition of D and K. Existence and uniqueness are still covered by the theory of ordinary differential equations, but now the "state space" is infinite dimensional so it is the classical theory of linear semigroups of operators that is applicable. System (S) still forms a building block for stability theory but no longer gives the full story. The added difficulties of dealing with residual and continuous spectra creep in. Other problems arise too. For example, even if the spectrum consists only of eigenvalues, what is one to say about the behavior of a system when all the eigenvalues λ_i, $1 \leq i < \infty$, of (S) satisfy $Re\lambda_i < 0$ but $Re\lambda_i \to 0$ as $i \to \infty$? In what sense is the rest state of the system asymptotically stable? These are definitely infinite-dimensional problems unknown in the finite-dimensional case (E).

Of course issues that were important in the finite-dimensional case continue to remain important. For example, if one block of the stiffness matrix K is much larger than the rest, multiplication of (E) by K^{-1} will yield a system with some components of the coefficient matrices for y, $\dot{y}$ being small. Not surprisingly such systems are termed "stiff" and their analysis is delicate. In infinite dimensions, the analogous situation may occur if two elastic bodies (one "stiff", one not) are attached or the mechanical stiffness of one body becomes very large in regions of the body. Also the *computation* of eigenvalues and eigenvectors for (S) is still of the utmost importance. The theory of spectral perturbation from known families of eigenvalues and eigenfunctions comes into play at this point: a serious analytical issue at both the finite- and infinite-dimensional levels.

The book under review brings together in a unified way the mathematical tools needed for a rigorous study of continuous systems in the spirit of the above discussion. While individual topics have been treated in other monographs, I know of no other place which provides such a nice conglomeration of mathematical theory and physical examples. (For example, while Kato's book [2] is a classic reference for spectral perturbation theory, linear semigroups, and linear functional analysis in general, it gives no guide to the researcher on how to "set up" a given continuum mechanical problem so as to avail oneself of the theory.) The emphasis is on analytical and abstract methods with operator theory, functional analysis, and the theory of linear partial differential equations in the forefront. Formal asymptotic methods are also treated (and quite well at that) though with less prominence. The physical examples are more or less ad hoc. The presumption is that the reader would use the book as a source for mathematical methods, not continuum mechanics.

A more detailed inspection of the contents yields: Chapter I, Classical Theory of Vibration for Systems with Infinitely Many Degrees of Freedom; Chapter II, some Classical Vibration Problems; Chapter III, Elements of Operator Theory; Chapter IV, Examples of Nonstandard Vibrations and Coupling; Chapter V, Spectral Perturbation; Chapter VI, Formal Perturbation Methods; Chapter VII, Perturbation of Vibrating Systems; Chapter VIII, The Helmholtz

Equation in Unbounded Domains; and Chapter IX, Scattering Problems Depending on a Parameter. Elastic Structure-Fluid Interaction in Unbounded Domains.

All in all, it is a lovely book and long overdue.

REFERENCES

1. G. Gallovotti, *Elementary mechanics*, Springer-Verlag, New York, 1983.
2. T. Kato, *Perturbation theory for linear operators*, Springer-Verlag, Berlin, 1966.

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An introduction to the numerical analysis of spectral methods, by Bertrand Mercier. Lecture Notes in Phys., Springer-Verlag, Berlin, New York, 1989, 154 pp., $23.30. ISBN 3-540-51106-7

Spectral methods form a relatively young and vigorously expanding field of numerical analysis. In the last 10 to 15 years, they have been applied to a wide variety of problems of mathematics and engineering. Concomitantly, the theoretical analysis of these calculations has grown and diversified, although, as usual in practical applications, we still compute much more than we can prove. The interested reader can do no better than consult [1], which surveys the state-of-the-art situation in the late 80s, giving ample coverage to both theory and applications.

The basic ingredient of spectral methods is the expansion of the unknown quantities in the series of orthogonal functions; these functions, in turn, result from the solution of a Sturm–Liouville problem. In practice, one considers either Fourier expansions—usually for periodic problems—or expansions in terms of orthogonal polynomials. Among the latter, the Chebyshev polynomials play a distinguished role, as they are amenable to the fast Fourier transform, but also admit more general boundary values than those allowed in Fourier series.

Consider now a typical differential problem $Lu = f$, for the unknown u. After u is replaced by an N-term expansion in the