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JOHN MCCARTHY
STANFORD UNIVERSITY

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Cosmology in $(2+1)$*-dimensions, cyclic models, and deformations of* $M_{2,1}$, by Victor Guillemin. Ann. of Math. Stud., vol. 121, Princeton University Press, Princeton, N.J., 1989, 227 pp., $50.00 ISBN 0-691-08513-7

A Riemannian (i.e., positive definite) metric on a compact manifold is called a *Zoll* metric if all of its geodesics are simply periodic with period 2π. The classic example of a Zoll surface is S^2 with the standard metric g_s. A number of years ago, Funk proposed the problem of finding all Zoll metrics on S^2 which are close to g_s. The underlying motivation of the present monograph is to consider a generalization of this problem of Funk to Lorentzian manifolds.

A metric g on the n-dimensional manifold M is said to be *Lorentzian* if it has signature $(+, \dots, +, -)$ at all points of M. One may denote the signature of (M, g) by referring to M as a $(k+1)$-dimensional manifold where $k = n-1$. For example, studying cosmology in $2+1$ dimensions may be thought of as investigating cosmological questions on three-dimensional Lorentzian manifolds.

The Levi-Civita connection ∇ and geodesics for a Lorentzian manifold (M, g) are defined in the same way as for a positive definite Riemannian manifold. In the Lorentzian case, there are three types of geodesics $\gamma\colon (a, b) \to M$ corresponding to $g(\gamma', \gamma')$ being always positive, negative, or zero. The *null* geodesics are the geodesics with $g(\gamma', \gamma') = 0$. It is an interesting fact that, up to reparameterization, the null geodesics are invariant under conformal changes. Guillemin calls a metric g on a compact manifold M a *Zollfrei* metric if all of its null geodesics are periodic. One may think of (M, g) as cyclic because each point (i.e., event) of the model gets replicated a countable number of times.

Since M is compact, M admits a Lorentzian metric iff the Euler characteristic of M is zero. Thus, for $n = 2$ and M orientable, one should consider $M = S^1 \times S^1$ with the standard metric $g_{\text{can}} = d\theta_1\, d\theta_2$. If (N, g) is an oriented Zollfrei manifold of dimension two, then there is a covering map $\pi: N \to S^1 \times S^1$ such that $\pi^*(g_{\text{can}})$ and g are conformally equivalent. It follows that all oriented Zollfrei two-folds are of the form R^2/L where L is a rational lattice subgroup of R^2 and the null geodesics may be taken as the lines parallel to the coordinate axes. In three dimensions, the corresponding classification is not as easy. It is not known in dimension three which manifolds may carry a Zollfrei metric. On the other hand, Guillemin has an interesting conjecture that a Zollfrei manifold of this dimension must be diffeomorphic to certain standard examples. One standard example is given by $S^2 \times S^1$ with the usual Lorentzian product metric $g_2 \oplus (-d\theta^2)$. Three other standard examples correspond to spaces which have $S^2 \times S^1$ as a double cover. Among these three is the conformal compactification of Minkowski three-space $M_{2,1}$.

One approach to the problem of Funk for $M_{2,1}$ is to consider all infinitesimal Zollfrei deformations of this manifold. Guillemin first shows that there are no formal obstructions to extending an infinitesimal deformation of $M_{2,1}$ to a true deformation and then reduces the extendibility question to estimates for what he terms the generalized x-ray transform. He finds that the solutions of the linearized Einstein equations on $M_{3,1}$ are in one-to-one correspondence with the infinitesimal cyclic deformations of $M_{2,1}$. Consequently, these infinitesimal deformations of $M_{2,1}$ may be interpreted as "free gravitons".

In the last part of this monograph, Guillemin considers the conformal d'Alembertian associated with Zollfrei metrics and obtains a number of important results. Among other things, he shows that if M is Zollfrei, the Floquet operators are of the form $e^{i\sigma} I + K$ where σ is an integral multiple of $\pi/4$ and K is a compact operator.

This is a very important monograph which not only answers a number of important questions, but also raises a number of new and interesting questions. It will serve as the foundation for many future research projects.

JOHN K. BEEM
UNIVERSITY OF MISSOURI