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IEKE MOERDIJK
MATHEMATISCH INSTITUUT
BUDAPESTLAAN 6
POSTBUS 800010
3508 TA UTRECHT
THE NETHERLANDS

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

0273-0979/90 $1.00 + $.25 per page

From cardinals to chaos: Reflections on the life and legacy of Stan Ulam, by Necia G. Cooper. Cambridge University Press, Cambridge, United Kingdom, 1989, 318 pp. $75.00 ($25.00 paper). ISBN 0-521-36494-9

The traditional western form of rendering homage to a distinguished scholar is the Festschrift. At its best, it provides valuable raw material for historians of the intellect by tracing the Master's ideas from their sources to their reworkings and elaborations. Also, the sensitive delineation of creative personalities by those who were close to them can instruct and amuse us. The value of such a volume depends on the originality of those ideas, and on how strong and how colorful that personality is.

The Festschrift dedicated to Ulam, under review, is intriguing on both counts. Born in 1909, Ulam made his reputation in the thirties by his precocity and by a series of unexpected results in set theory, topology, measure theory, and functional analysis. These studies, and the scintillating quality of his mind, earned him the admiration and friendship of many mathematicians, especially of von Neumann, who was destined to play a decisive role in Ulam's life. It was von Neumann who brought Ulam to Los Alamos in 1944; that experience of being part of a vast engineering enterprise in which physics and mathematics played such decisive roles has profoundly altered the scientific outlook of both men. Each turned his back on the mathematical paradigm prevailing in the mid forties that placed the highest value on the study of elaborate structures, using methods that were technically extremely intricate; von Neumann partly because he found doing technical mathematics too easy, and Ulam because he found it too hard. What was more important, they both found the lure of brand new problems arising in physics, in biology, and in neurophysiology irresistible. Ulam shared von Neumann's vision that "high-speed computing devices may, in the field of nonlinear partial differential equations, as well as in many other fields which are now difficult or entirely denied of access provide us with those heuristic hints which are needed in all parts of mathematics for genuine progress." Ulam took the lead in carrying out such a program in a most imaginative way.

The volume before us gives a brief tour of Ulam's scientific output in the form of a series of essays labelled *The Ulam Legacy.* The tone of these essays is light; they are not technical, but they are not devoid of mathematics, either; they can serve as an introduction to their subject for someone previously unfamiliar with it. The mathematics is interspersed with observations of Ulam's way of thinking.

David Hawkins describes his collaborations with Ulam on branching processes, arising in nuclear fission and also in the study of population dynamics; the mathematics here is neoclassical probability.

Dan Mauldin writes about his work with Ulam on random nonlinear maps; he starts with the axioms of probability, and ends with invariant measures, attractors and repellers, Hausdorff dimension, and random homeomorphisms. I highly recommend this article to everyone, both for the exciting mathematics it contains, and for exemplifying "Stan's wild ideas ... that were the joy of being with Stan Ulam. His boundless imagination opened up one's mind to the endless possibilities of creating."

Paul Stein describes his work with Ulam on quadratic and cubic maps of a triangle into itself. They found that some cubic maps upon iteration behave chaotically, and that others converge to peculiar looking sets that were later dubbed strange attractors. This striking study from the late fifties did not attract much attention; the reason may be that, unlike later work of Henon and of Lorenz, it was not linked to equations of mathematical physics.

Jan Mycielski gives a capsule summary of Ulam's early work on measurable cardinals, his result with John Oxtoby on ergodicity, the Borsuk–Ulam theorem, and a surprising result on genealogical distance in a stationary population.

Ron Graham discusses Ulam's notion of similarity of two graphs with the same number of edges, defined as follows: Decompose each graph into r subgraphs so that each subgraph in one decomposition is isomorphic to a corresponding subgraph in the other decomposition. The smallest number r for which this is possible gives a measure of similarity of the two graphs. Several interesting theorems are described; applications lurk in the wings.

Nick Metropolis' account of the beginning of the Monte Carlo method has the intimacy of a story told by one who was there. The collaboration of Ulam, von Neumann, and Richtmyer in making Monte Carlo a reality is described by Rober Eckhardt; Doolen and Hendricks describe later applications, par excellence the uses of the Metropolis algorithm. As measure of the ubiquity of the Monte Carlo methodology, they mention that every second nearly 10^9 pseudo-random numbers are generated in the world for use in Monte Carlo calculations—a figure derived no doubt by statistical sampling. A brief account of the various ways to generate pseudorandom sequences is given by Tony Warnock.

Frank Harlow is one of the founding fathers of computational fluid dynamics; his unconventional ideas were encouraged by Ulam. Here he presents a kind and gentle account of the work he and members of his group have done over the past 18 years to calculate various regimes of turbulent flows.

Brosl Hasslacher's article describes recent attempts to use so-called lattice-gas automata for calculating flow patterns, in particular solutions of the Navier–Stokes equation. This approach deals with a discretized version of statistical mechanics reminiscent of the Broadwell model; what makes it attractive is the possibility of very efficient implementation. There are intriguing analytical difficulties, e.g., it is not clear if the solutions constructed here approach exact solutions of the Navier-Stokes equations or are merely good approximations to them; if the latter, how good? At any rate, the method is unsuitable for calculating flows with nontrivial thermodynamic effects, i.e., compressible flows. The philosopy of this interesting approach harks back to one taken by von Neumann in 1944 and then abandoned, and to Ulam's particle and force method from the late fifties.

David Campbell gives a sweeping review of what is nowadays called nonlinear science but was formerly called merely science. He focuses on three paradigms: *coherent structures and solitons, deterministic chaos and fractals, and complex configuration and patterns*. Coherent structure is not a precisely defined concept; it fits Justice Powell's characterization of pornography: "I recognize it when I see it." Solitons, discovered by Kruskal and Zabusky, are the stablest of the coherent structures; they retain their shapes through thick and thin. Others, such as vortices, shock waves, surface waves, bubbles, flame fronts, etc., are ephemeral.

Deterministic systems, such as the ones governed by classical ODE's, have the property that their state at any time t is uniquely determined by their initial state. But many deterministic systems appear to behave in the long run in an irregular, unpredictable, random fashion; this phenomenon is called deterministic chaos. Poincaré put his finger on the origin of deterministic chaos: "Small differences in the initial conditions produce very great ones in the final phenomena... prediction becomes impossible and we have fortuitous phenomena."

Examples of deterministic chaos abound; ergodicity, the brainchild of statistical mechanics, but for many years a branch of mathematics, is a striking example. In the early part of this century, Fatou and Julia carried out deep investigations on the iterates of quadratic maps in the complex plane, and found unexpectedly complicated behavior. Another low-dimensional case appears in the studies of electronic devices by Littlewood and Cartwright in the 1940's. In 1960 Sitnikov showed that the restricted three-body problem has solutions that have the appearance of being random. An even more profound manifestation of deterministic chaos may be turbulence in fluid dynamics, whose understanding has been one of the outstanding goals of mathematical physics for more than a century.

Many other examples from the past could be added; we mention one on the discrete level: The discovery by Kac and Erdos that many number theoretic sequences appear to be random.

The examples cited above show that deterministic chaos has been under investigation for almost a hundred years by some of the outstanding mathematicians of the century. It is therefore quite incorrect to say that "deterministic chaos remained virtually unexplored and unknown until the early 1960's." But it is absolutely right to say that computing, especially interactive computing, and good graphics, have enormously advanced research in this area. We can for the first time glimpse the enormous complexity of chaotic regimes, interspersed with patterns of regularity, and thereby receive heuristic hints which enable the prepared mind to formulate theorems and theories. Campbell gives thumbnail descriptions of several well-known instances, and a few not yet so well known. Among the former is the behavior of the logistic map under iteration when the control parameter is varied, resulting in doubling of the periodic attracting set until a critical value is reached; beyond it the iterates exhibit chaotic behavior. Feigenbaum's astonishing discovery of universality concerning this period doubling is briefly described. The next three examples are not so well understood theoretically: the damped, driven pendulum; the Lorenz attractor; and the so-called standard map of a two-dimensional torus onto itself, preserving area. Among the less well-known examples are the Kuramoto–Sivashinsky equation, the damped, driven two-dimensional sine-Gordon equation, and the Kolmogorov–Spiegel–Sivashinsky equation. The notion of fractal is described and the fractal nature of attractor sets is discussed in both theoretical and experimental settings. All these phenomena are richly illustrated by multicolored computer graphics and photographs of physical experiments.

The article by Adrian Patrascioiu deals with implications for physics of the failure of the ergodic hypothesis, such as occurs in the numerical experiment of Fermi–Pasta–Ulam, and for Hamiltonian systems that satisfy the hypotheses of the KAM theory. The author raises the possibility that for such systems the distribution of energy might follow Planck's law. That is, the Planck distribution is derivable from nonlinear classical mechanics!

Ulam was fascinated by the role mathematics may play in biology and physiology. Some of his ideas were expressed in his Gamow Memorial Lecture held in 1982, published here for the first time. It deals with various notions of distance between natural objects. One suggested application of such a notion is to explain how the brain manages to recognize patterns so unerringly. Another possible use is to measure the distance between the genetic codes of cognate species. An interesting article by Walter Goad explains the basic facts of modern molecular genetics and possible uses of Ulam's ideas in comparing DNA sequences.

The volume concludes with a number of off-beat philosophical discussions between Ulam and Rota on the nature of mathematics skillfully transcribed from tape recordings.

I turn now to the remarkable essay entitled, "The Lost Cafe" written by Gian-Carlo Rota, that serves as introduction to this volume. Rota had known Stan Ulam for the last twenty-one of his seventy-five years, as Boswell had known Samuel Johnson. The essay is a kaleiodoscopic picture, filtered through Rota's eyes and ears, of Ulam's life and thoughts, his psychological makeup, the influence of his European background and the destruction of Europe during the Second World War, his friendship with von Neumann, and the thrill of being part of the great scientific enterprise at Los Alamos.

Rota claims that a severe case of encephalitis in 1946 that nearly took Ulam's life impaired his ability to do technical mathematics. Rota did not know Ulam before 1946; most of those who did disagree with this judgement, and think that Ulam's inability or unwillingness to do calculations was innate and went back to his youth. I recall Ulam describing his suffering in a course on descriptive geometry he had to take as a young student in Lwow where he was required to make precise mechanical drawings. After squeaking through the course—no doubt with the lowest passing grade—Ulam had his revenge on the professor who was his tormentor, by telling him: "Poincaré said that geometry is the science of correct reasoning from incorrectly drawn figures; for you it is the other way around."

In the end it doesn't much matter why Ulam shied away from technical mathematics; what matters is that we see him as he was, a very human hero who succeeded in turning a weakness into major strength. Rota's scientific and psychological portrait, suffused with love, pain, understanding, and admiration, succeeds in bringing him to life.

The unusual and unusually beautiful design and artwork, including three portraits of Ulam by Jeff Segler, enhance the value of this volume and the pleasure it gives; the editor, Nicky Cooper, has earned our gratitude.

PETER D. LAX
NEW YORK UNIVERSITY
COURANT INSTITUTE

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 22, Number 2, April 1990

0273-0979/90 $1.00 + $.25 per page

Kleinian groups, by Bernard Maskit. Grundlehren der Mathematischen Wissenschaften, vol. 287, Springer-Verlag, Berlin, Heidelberg, New York, 1988, xiii + 326 pp., $77.50. ISBN 3-540-178746-9

Colleagues and friends of Bernie Maskit can turn to almost any page of his book and immediately recognize his characteristic style. The focus is on that part of the field where his own contributions are most strongly felt. Before turning to the book itself, we will make some general comments.

In the complex analysis we learn early about Möbius transformations; the conformal automorphisms of the (Riemann) 2-sphere; and their classification into elliptic, parabolic, and loxodromic/hyperbolic. Each is the composition of two or four reflections in circles which following Poincaré leads easily to its natural extension to a conformal automorphism of the 3-ball. Besides giving rise to the orientation preserving conformal automorphisms of the ball, this procedure also establishes that the totality of these extensions gives the full group of orientation preserving isometries of hyperbolic 3-space $\mathbf{H}^3$.