everybody although with an emphasis on problems involving jets and cavities. Troianiello, on the other hand, has a more focused point of view in a very different direction. He develops the classical theory of elliptic differential equations and regularity of their solutions in the framework of variational inequalities. A lot of high powered technical machinery is used, and this book is the most theoretical of those listed here. Rodrigues points out in the preface that he is trying for a modern version of Duvaut and Lions; he is more concerned with physical motivation than with development of theory. Theorems on elliptic differential equations are quoted as needed. The book succeeds at emphasizing the physical point of view without disregarding mathematical rigor. Some of the models are described rather sketchily, though.

In addition to the applications, Rodrigues spends more time on the study of stability of free boundaries then the other authors listed.

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Nest algebras, by Kenneth R. Davidson. Pitman Research Notes in Mathematical Sciences, vol. 191, Longman Scientific and Technical, Harlow, 1988, 411 pp., $74.95. ISBN 0-582-01993-1

Nest algebras were introduced by J. R. Ringrose in 1965 shortly after studies by R. V. Kadison and I. M. Singer of a related class of operator algebras, and in the last twenty-five years the subject has matured to the extent that they form a moderately well-understood class in the category of non-self-adjoint operator algebras. Most notably Ringrose's similarity problem has been resolved, finally, in a curious way requiring a deep and unexpected excursion into the analysis of quasitriangular algebras. Moreover there are now multiple points of contact with other areas of operator theory and many intriguing basic problems remain unsettled.

"Non-self-adjoint." This is, unfortunately, a rather inelegant adjective, a kind of apologetic antidefinition, but it may be seen less in the coming years, both because of the widening concerns of general operator algebras and because of the strengthening identity of special algebras such as nest algebras. Let me say at once that Davidson's beautiful book is an important comprehensive account of nest algebras and related areas, with twenty five chapters embracing the basics, very recent research, and a great deal of what has gone inbetween, much of it of the author's own creation.

So what are nest algebras? The stock answer, that they are the Hilbert space generalisations of the algebra of upper triangular complex matrices, is admittedly somewhat unexciting, but it evokes some thoughts that turn out to be far-reaching; there should be close connections with upper triangular forms and single operator theory, and, in view of the nontriviality of the Jacobson radical (strictly upper triangular matrices in the matrix case), there should be an interesting ideal theory. The latter thought has a reality in Ringrose's seminal papers, where the Jacobson radical is characterised. It is also connected, in an indirect way, with the more recent resolution of the similarity problem, which I will now sketch.

A subspace nest $\mathcal{N}$ is a totally ordered family of closed subspaces of a fixed complex Hilbert space, which is complete in a natural sense. The associated nest algebra $T(\mathcal{N})$ consists of all operators which leave invariant each subspace N of $\mathcal{N}$. Nests $\mathcal{N}$ and $\mathcal{M}$ are similar if $\mathcal{M} = \{SN : N \in \mathcal{N}\}$ for some invertible operator S between the underlying Hilbert spaces. (Equivalently $T(\mathcal{M}) = ST(\mathcal{N})S^{-1}$.) Important examples of nests are obtained from Borel measures μ on the unit interval: $\mathcal{N}_\mu = \{N_t, N_{t+} : 0 \leq t \leq 1\}$ where N_t and N_{t+} are the subspaces of functions supported on $[0, t)$ and $[0, t]$ respectively. Lebesgue measure gives the Volterra nest, so called because its algebra contains the Volterra integral operator

$$Vf(x) = \int_x^1 f(t)\,dt.$$

This particular nest is continuous both in the measure-theoretic sense (no atomic masses) and, more aptly, in the nest-theoretic sense that no subspace of the nest has an immediate successor. Up to unitary equivalence general nests are (almost) completely understood in terms of J. A. Erdos' spectral theorem which identifies a triple of order type, measure type, and multiplicity type as the complete set of invariants. The examples $\mathcal{N}_\mu$ are in fact the typical multiplicity one nests on a separable Hilbert space. The similarity problem asks for an analogous understanding with regard to similarity in place of unitary equivalence. The importance of the problem lies in the fact that such an understanding would lead to a classification of nest algebras up to similarity, and, indeed, also up to algebraic isomorphism. However, for many years the most basic questions lay open. Can similarity change multiplicity? Is the Volterra nest similar to the analogous multiplicity n nest on $L^2[0, 1] \otimes \mathbf{C}^n$?

The answers to these questions (Yes! Yes!) were as curious as the methods needed to obtain them. In his thesis (circa 1980) N. T. Andersen, one of W. B. Arveson's many non-self-adjoint students, showed that two quasitriangular algebras $QT(\mathcal{N})$, $QT(\mathcal{M})$ are unitarily equivalent if their nests are approximately unitarily equivalent, and furthermore, that this condition prevails for any pair of continuous nests on separable Hilbert space. Here $QT(\mathcal{N})$ is the algebra $T(\mathcal{N}) + \mathcal{K}$ obtained by adding the ideal of compact operators. With this important work D. R. Larson was able to settle the multiplicity problem in a strong way by showing that all continuous separable nests are similar. Finally, through a deeper understanding of approximate unitary equivalence, Davidson showed that the (obvious) invariant of order-dimension type provided the complete invariant for similarity. Subsequently, similarity theory, as it is now known, has led to a number of interesting results such as, the triviality of the commutator ideal of a continuous nest algebra, the identification of the outer algebra automorphism group with the group of dimension-preserving order homomorphisms of the nest, an analogous identification in the case of the quasitriangular algebras (Davidson-Wagner), the calculation of K_0 (D. R. Pitts), and, quite recently, the unexpected maximality, as a diagonal disjoint ideal, of Larson's ideal $R^{\infty}(\mathcal{N})$ (J. L. Orr).

The comprehensive nature of Davidson's book makes it an essential guide for graduate students and others becoming interested in general operator algebras on Hilbert space, and it also serves as a very useful reference for specialists. Similarity theory proper occupies little more than two twenty fifths of the book. There is a somewhat modernised treatment of compact operators and triangularity in part I (Chapters 1–5), including classical results of Matsaev, Gohhberg and Krein, and a section on the Volterra operator. Part II (Chapters 6–13) concentrates on basic structure theory; the radical, unitary invariants, M-ideals and distance formulae, and similarity theory, whilst part III (Chapters 14–21) develops additional topics; factorisation, ideals and bimodules, duality, isomorphisms, perturbation theory, derivations, representation and dilation theory, unitary orbits of nest algebras, and results on unicellular operators and attainable lattices. Finally part IV (Chapters 22–25) constitutes an introduction to Commutative Subspace Lattices and their CSL algebras. These are natural generalisations of nest algebras in which the invariant subspace lattice is commutative (meaning that the family of the self-adjoint subspace projections is commutative) but not necessarily totally ordered. Davidson gives a fresh look at Arveson's fundamental work (1974) in this area, including the difficult technicalities concerning synthetic subspace lattices, as well as discussions of complete distributivity and J. Froelich's recent and striking exploitation of classical harmonic analysis in the construction of nonsynthetic lattices and CSL algebras with prescribed properties. The final section discusses ten basic open problems. Also, each of the twenty four main sections closes with brief notes providing attribution of the results, and with a collection of well-chosen problems complementary to the text.

The book is extraordinary thorough, with full details of a very wide range of contemporary topics, some as yet unpublished. Many specialists will be pleased that Davidson has brought their own interests to a wider audience so lucidly.

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