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JON F. CARLSON
UNIVERSITY OF GEORGIA

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

Univalent functions and Teichmüller spaces, by Olli Lehto. Graduate Texts in Mathematics, vol. 109, Springer-Verlag, New York, Berlin, Heidelberg, London, Paris, New York, 1987, xii+257 pp., $46.00. ISBN 0-387-96310-3

Quasiconformal mappings have played a prominent role in geometric function theory for nearly fifty years. We recall that a sense-preserving diffeomorphism f of one plane region D onto another is called a K-quasiconformal map if its differential $f'(x)$, viewed as a linear map of $\mathbf{R}^2$ onto itself, satisfies the inequality

$$\max\{\|f'(x)u\|; \|u\| = 1\} \le K \min\{\|f'(x)u\|; \|u\| = 1\}$$

at every point in D. A sense-preserving homeomorphism of D into the plane is K-quasiconformal if there is a sequence of K-quasiconformal diffeomorphisms that converges to f uniformly on compact subsets of D. A sense-preserving homeomorphism of one Riemann surface onto another is K-quasiconformal if all its compositions with local charts are K-quasiconformal maps in the plane. Finally, a quasiconformal map is a sense-preserving homeomorphism that is K-quasiconformal for some number $K \ge 1$. It is important to observe that 1-quasiconformal maps are conformal.

In 1939, a fundamental paper of Teichmüller introduced quasiconformal maps to the study of spaces of Riemann surfaces. Choose a compact Riemann surface X of genus $p \ge 2$ and define a pseudometric on the set of all sense-preserving homeomorphisms of X onto Riemann surfaces of the same genus by putting

$$d(f,g) = \log K \tag{1}$$

if K is the smallest number such that there is a K-quasiconformal map in the homotopy class of $g \circ f^{-1}$. The metric space that results from identifying f with g when $d(f,g) = 0$ is Teichmüller's space T_p of marked Riemann surfaces of genus p. Teichmüller proved that T_p is homeomorphic to $\mathbf{R}^{6p-6}$.

From the 1950s to the present day, the theory of Teichmüller spaces has been actively developed, primarily by Ahlfors, Bers, Rauch, and their students. A decisive step was taken by Bers in the 1960s. If X is as above and $f\colon X \to Y$ is a quasiconformal map, we can choose holomorphic universal coverings of X and Y by the open unit disk Δ and lift f to a homeomorphism $f^{\#}$ of Δ onto itself. There is a unique quasiconformal map w of the plane onto itself such that $w \circ (f^{\#})^{-1}$ is conformal in Δ and w is conformal in the exterior Δ^* of Δ, with behavior

$$w(z) = z + O(z^{-1})$$

at infinity. Bers associated to f the Schwarzian derivative $\varphi = S(w)$ of the conformal map w in Δ^* and showed that the assignment $f \mapsto \varphi$ induces a well-defined injective map on T_p.

Bers's construction focused attention on the class of conformal maps of Δ^* into the Riemann sphere that admit quasiconformal extensions to the sphere. These are an interesting class of univalent functions, whose properties have been extensively studied. The image of the unit disk under such a mapping is called a quasidisk, and the geometric and analytic properties of quasidisks have also been much investigated. There has thus been an active interplay between the theory of Teichmüller spaces and other branches of geometric function theory.

This brings us to Lehto's book. The author is a distinguished geometric function theorist who has already co-authored the standard book [**6**] on plane quasiconformal maps. His is one of three new books (the others are [**3** and **7**]) that for the first time offer a detailed comprehensive treatment of the complex analytic properties of Teichmüller spaces for the general mathematical reader. All of them are welcome. The distinctive feature of Lehto's book is its emphasis on the interplay between Teichmüller space theory and univalent function theory. Indeed the first half of his book is devoted to a very lucid treatment of quasidisks, univalent functions with quasiconformal extensions to the sphere, and the universal Teichmüller space, which is simply the space of Schwarzian derivatives of such univalent functions. This is an interesting branch of geometric function theory, to which the author has made substantial contributions, and his treatment of this material is a delight to read.

The remainder of the book consists of an expository chapter that surveys necessary background material about Kleinian groups, Riemann surfaces, and the geometry of quadratic differentials, followed by a final 73 page chapter on Teichmüller spaces. The quality of the exposition remains high and, considering that very little is taken for granted, it is remarkable how much material is covered in that final chapter. It includes a complete proof, making shrewd use of Bers's Schwarzian derivative map $f \mapsto \varphi = S(w)$, that every Teichmüller space is a complex manifold (not always finite dimensional), a discussion of extremal quasiconformal maps, using the Hamilton-Krushkal' approach, and finally a complete proof of Teichmüller's theorem about the extremal mappings in T_p.

Every reader who enjoys geometric function theory will want to own this book. Readers who want to learn about Teichmüller spaces will find this book a very good beginning but will also want to look at the two other new books on the subject and at some of the additional references listed below or in the excellent bibliography at the end of Lehto's book.

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CLIFFORD J. EARLE
CORNELL UNIVERSITY

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 19, Number 2, October 1988

0273-0979/88 $1.00 + $.25 per page

Kleinian groups and uniformization in examples and problems, by S. L. Krushkal′, B. N. Apanasov, and N. A. Guseviskiĭ. Translated from Russian by H. H. McFaden. Translations of Mathematical Monographs, vol. 62, American Mathematical Society, Providence, R.I., 1986, vii+198 pp., $66.00. ISBN 0-8218-4516-0

The old order changes; classical divisions of mathematics into subject areas of distinguishable type have been progressively refined and fragmented until the attempt to classify a research paper via the MR subject index appears as a task of comparable size to understanding the results themselves. This Balkanisation process is compounded by an increasing—and no doubt welcome—tendency towards federalisation of the ideas and techniques which erodes and transcends even the ancient divides of algebra, analysis, and geometry.

How, for instance, should one approach Kleinian groups? As discrete subgroups of the Lie group of complex two-by-two matrices, Kleinian groups fit naturally within at least four broad subject areas, reflecting their origins within the classical analysis, the underlying (abstract) group-theoretical structures which they represent, their position within the deformation theory of discrete groups in general, and the topological connection with hyperbolic three-dimensional manifolds first noticed by Poincaré and recently brought back to prominence by Thurston's revolutionary ideas. None of which mentions the specific and important links with number theory, algebraic groups, the geometry of algebraic curves and their moduli spaces, or the analogy with