9. H. P. McKean, *Brownian motion with a several dimensional time*, Theor. Probab. Appl. **8** (1963), 335–354.

10. L. D. Pitt, *A Markov property of Gaussian processes with a multidimensional parameter*, Arch. Rational Mech. Anal. **43** (1971), 367–391.

11. S. O. Rice, *Mathematical analysis of random noise*, Bell System Tech. J. **24** (1945), 46–156.

12. I. J. Schoenberg, *Metric spaces and completely monotone functions*, Ann. of Math. (2) **39** (1938), 811–841.

13. I. J. Schoenberg, *Positive definite functions on spheres*, Duke Math. J. **9** (1942), 96--108.

14. M. Weber, *Une méthode élémentaire pour l'étude de la régularité d'un large classe de fonctions aléatoires*, C. R. Acad. Sci. Paris Ser. A **292** (1981), 599–602.

SIMEON M. BERMAN

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 13, Number 1, July 1985
© 1985 American Mathematical Society
0273-0979/85 $1.00 + $.25 per page

Modern geometry (*Sovremennaya geometriya*), by B. A. Dubrovin, S. P. Novikov, and A. T. Fomenko, Moskva:Izdatel'stvo "Nauka", 760 pp., R.2.00 (Russian), 1979.

This is a very important and interesting book. This volume is written by Russian authors who are experts in large areas of contemporary geometry and their applications. In particular, Novikov is a Field's Medalist and is publishing most exciting research in the intersection of geometry, analysis, and physics. The original Russian edition appeared in 1979 in a one-volume edition, double the size of the present translated volume. I first came across this book shortly thereafter when a new faculty member in our department, Alexander Eydeland, showed it to me. In 1982 at the colloquium in Paris to honor Laurent Schwartz, I found a translation of this book in French, in two volumes. The present English translation comprises the first volume of the French edition.

The best way to describe this volume is to say that it is a contemporary treatise on modern methods in geometry with deep applications to the physical sciences. It used to be that the great treatises of mathematics were written by scholars of analysis, and these volumes contained a synthesis of the geometry and analysis of their times. The latest example of this type of work is the comprehensive and marvelous treatise on analysis by Jean Dieudonné in twenty-five chapters, twenty-four of which have appeared in nine volumes.

As a student, I studied geometry for each of my undergraduate years, but unfortunately it was not the kind of geometry presented in this new Russian book. Indeed, it was filled with linear projective geometry in all its classical points of view (triangles, lines, hyperplanes, crossratios, and eventually conic sections). (Even Euclid would have felt maligned.) Eventually I escaped to England to discover that projective geometry could include cubic surfaces and, indeed, surfaces of even higher degree. Unfortunately, this was never connected with the other interesting aspects of differential geometry and contemporary science. With the present Russian volume, this tradition is, I hope, over, and students will be liberated from the terrible artificial boundaries and clumsy fences that have been placed in the garden of mathematics and its applications to science.

Moreover, the present book sets out in an extremely ambitious and somewhat novel direction: namely, to make geometry the fundamental unifying field for contemporary mathematical thought and research. Thus, in this sense, the book is truly "modern". What is doubly interesting is that the volume can be read by Russian or American undergraduates with some prior mathematical specialization. The style is extremely concrete, excellent examples illustrate the material discussed, and interesting formulas and lovely pictures fill the pages. This volume is, indeed, not written in the Bourbaki tradition, and so it symbolizes, along with the splendid volumes of V. Arnol′d, a new era in mathematical style, albeit with a melodic Russian intensity.

The present English translation is divided into six chapters, and here is a summary of some of the highlights. The first chapter discusses the simplest transformation groups with numerous examples, the Serret-Frenet formulae, and how these ideas unify the geometric concepts of the special theory of relativity, together with a section on Lorentz transformations. All in all, this chapter is a tremendous intuitive tour de force of clear exposition.

Chapter two is a discussion of the elementary geometry of real and complex surfaces, including a discussion of Riemannian metrics, the second fundamental form, Gaussian curvature, Hermitian scalar products, surfaces of constant Gauss curvature, and transformation groups as surfaces. The first two chapters do not involve much unusual notation beyond simple notations for matrices and quaternions. However, they develop a great deal of insight into the geometrical ideas involved without subdividing mathematics into the straightjacket of analysis, algebra, geometry, and science.

Chapter three is devoted to tensors and differential forms, together with interesting information on the crystallographic groups and their finite subgroups. This is illustrated by concrete pictures and explicit, interesting examples. Halfway through the chapter there is a discussion of the invariant properties of the electromagnetic field based on skew-symmetric tensors and Maxwell's equations. This illustration is a remarkable, but typical, example of this book's great taste. The chapter ends with a discussion of Lie algebras, Killing metrics, and vector fields.

Chapter four discusses the differential and integral calculus of tensors. In particular, there is a good discussion of the integration of differential forms on a manifold, together with a general form of Stokes' formula and its relevance to Maxwell's equations. There is also a very good discussion of covariant differentiation and connections on a manifold with both the real and complex structures. The end of the chapter is involved in a discussion of the curvature in higher-dimensional cases.

Chapter five is the pièce de résistance, including as it does, the elements of the calculus of variations in its relationship to all of the previous material. This selection of material is marvelous, because it is the key unifying scheme that has been used in both mathematics and physical science in the present century. The discussion given in the book starts off slowly with the Euler-Lagrange equations, and then moves on to conservation laws and their connection with transformation groups for a given variational problem. Both the Hamiltonian and Lagrangian formalisms of classical mechanics, Poisson brackets, and gradient systems are discussed, along with much other information.

Chapter six continues this treatment of the calculus of variations, but moves to higher-dimensional examples. The applications are marvelous in this section, focusing as they do on those coming from the physical sciences. Thus, Maxwell's equations as well as Einstein's general theory of relativity are discussed. Elasticity is mentioned, as well as the Dirac equations of quantum mechanisms, together with their associated ideas of spinors as representations of the Lorentz group.

The book ends by discussing gauge-invariant lagrangians, and in this way unifies the ideas of geometry, physical science, and calculus of variations. Yang-Mill's equations are discussed, as well as Maxwell's equations, as examples of the theories developed.

It is customary for a reviewer to add some negative points about each book he reviews. Although I very much like this book, a few points might be mentioned in the text. One of the great goals of the modern approach to geometry is to restore analysis to its former unique position in the geometrical hierarchy. In other words, one would like to put the word "differential" back into differential geometry. This book falls a little short in this connection. When a nonlinear problem arises, the authors find explicit solutions for it, or else leave the topic. The structures of analysis, based on limits of sequences solving ordinary and partial differential equations, and clever uses of Hilbert spaces and integration are not emphasized. The analysis associated with the Laplace-Beltrami operator and its eigenvalues is not clarified, and the roles of Sobolev spaces and isoperimetric inequalities are not mentioned. My own prejudice would be to add a little of the basic nonlinear phenomena that enter into the subject. On the other hand, the book cannot do everything in 454 generously spaced pages.

Certainly, with this type of material under their belts, any student or researcher will be prepared to enter the exciting world of the twenty-first century, in which many mathematical areas that were thought to be distinct are now being shown to be part of a great unity.

This is, of course, only one-half of the Russian volume, so what could be contained in the second half? We certainly look forward to the English translation of this part, and, to encourage any anxious reader, let me say Volume two is even more exciting than the present one. It starts off with a discussion of manifolds, both real and complex, discusses the basic analysis on such varieties, and eventually ends with a very clear, detailed discussion of both general relativity and Yang-Mill's equations.

Physics is distinguished by an important tradition of book writing. Some of its finest adherents have published comprehensive overviews of the entire subject displaying their own personal tastes and scientific personalities. These volumes range from Feynman's three, to Arnold Sommerfeld's seven, to Landau's and Lifschitz's at least ten. Let us hope that this volume, with its incisive vision of the unity of mathematics, will initiate a similar fashion in the mathematical community. I believe such book writing is long overdue.

MELVYN S. BERGER

BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 13, Number 1, July 1985
© 1985 American Mathematical Society
0273-0979/85 $1.00 + $.25 per page

Extremes and related properties of random sequences and processes, by M. R. Leadbetter, Georg Lindgren and Holger Rootzen, Springer Series in Statistics, Springer-Verlag, New York, Heidelberg, Berlin, 1983, xxi + 336 pp., $36.00. ISBN 0-387-90731-9

The classical theory of extreme values of probability theory deals with the asymptotic distribution theory of the maxima and the minima of independent and identically distributed (i.i.d.) random variables. That is, let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with common distribution function $F(x)$. Put $W_n = \min(X_1, X_2, \ldots, X_n)$ and $Z_n = \max(X_1, X_2, \ldots, X_n)$. Then the distribution functions of W_n and Z_n satisfy

$$L_n(x) = P(W_n \leqslant x) = 1 - [1 - F(x)]^n$$

and

$$H_n(x) = P(Z_n \leqslant x) = F^n(x).$$

It is rare in probability theory that $F(x)$ is known. Indeed, the field of determining $F(x)$ from some elementary properties, known as characterizations of probability distributions, is quite recent (for the history of the field of characterizations, see the introduction in Galambos and Kotz (1978)). On the other hand, if $F(x)$ is determined by some approximation, however accurate, the values of $H_n(x)$ and $L_n(x)$ cannot be computed from the above formulas due to the sensitivity of u^n to u for large n (compare $0.995^{400} = 0.1347$ and $0.999^{400} = 0.6702$). This difficulty is overcome in an asymptotic theory that is invariant for large families of population distribution $F(x)$. In other words, for varying $F(x)$, linearly normalized extremes $(Z_n - a_n)/b_n$ or $(W_n - c_n)/d_n$ have the same limiting distribution functions $H(x)$ or $L(x)$, respectively. The theory is well developed for finding these appropriate normalizations and the forms of the limiting distribution functions, as well as for easy-to-apply criteria for $F(x)$ leading to a particular $H(x)$ or $L(x)$. Chapter 2 of Galambos (1978) gives a full account of this theory.

The classical theory of extremes can at best be applied as a first approximation to real-life models. Observations collected in, or produced by, nature are rarely independent, and neither are components of pieces of equipment functioning independently. For example, floods, defined as the highest (random) water level of a river at a given location, are clearly obtained through strongly dependent values, which dependence might weaken as time goes on. In