The number of minor errors in the book is annoyingly large. Sometimes the proofs have the appearance of having been hastily written and then not carefully checked. Some sections are sufficiently sloppy to cause confusion. In spite of these problems, there was no lack of volunteers to lecture from the book at a year long seminar at Michigan State University. We covered almost the entire book and everyone was enthusiastic about the choice of topics that Fisher made. For anyone interested in complex analysis, there is a lot of fun (and a bit of frustration) to be found in this book.

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Integral representations and residues in multidimensional complex analysis, by L. A. Aĭzenberg and A. P. Yuzhakov, Translations of Mathematical Monographs, Vol. 58, American Mathematical Society, 1983, x + 283 pp., $68.00. ISBN 0-8218-4511-X

One of the most beautiful and, at the same time, useful portions of mathematics is the classical theory of the Cauchy integral: the Cauchy integral theorem and the residue theorem. This theory has found application in remarkably diverse directions, from number theory to hydrodynamics. In addition to such extramural applications, the Cauchy theory has, of course, played an essential role in the development of the magnificent edifice that is the modern theory of functions of a complex variable.

The development of the theory of functions of several complex variables, beginning with Riemann and Weierstrass, has followed rather different lines. In this development the Cauchy integral formula of classical function theory has played an important role, for example, in the usual proof of the Weierstrass preparation theorem. In addition, there is an immediate extension of the Cauchy integral formula, valid for functions holomorphic on polydiscs, and it has important uses, which, in the main, parallel the simpler uses of the one-dimensional theory. However, integral representation formulas of an essentially multidimensional nature have not played a major role in the development of the theory of functions of several complex variables. Thus, the two standard English language texts on several complex variables, Gunning and Rossi's *Analytic functions of several complex variables* [6], and Hörmander's *An* *introduction to complex analysis in several variables* [**9**], which appeared in the mid-sixties, devote very little space to integral formulas; one finds in them essentially nothing beyond the one-dimensional Cauchy integral formula.[1]

This apparent gap cannot be attributed to an absence of results on integral formulas and residue theory in many variables. There was from the late 1880's until the late 1960's a gradual accretion of information in this direction. Some of the major contributions to this development were Poincaré's memoir of 1886 [**14**], which considered periods of holomorphic differentials on algebraic varieties, the development of the theory of kernel functions, particularly in the hands of Stefan Bergman (see [**1**]), and Weil's construction [**16**] of integral formulas on analytic polyhedra. Another important development was the Bochner-Martinelli integral formula published first by Martinelli (see [**13**]) in the late thirties and then by Bochner [**2**] in the early forties. Finally, we should mention the memoir of Leray [**12**], which contains a general theory of residues in n dimensions.

In spite of these contributions, each very substantial, the theory of integral formulas did not occupy a central position in the theory of functions of several complex variables as it was practiced through the sixties; it was not unreasonable for Gunning and Rossi and for Hörmander to omit serious treatments of the subject from their texts. At the end of the sixties, matters changed. As the cause of this change, we should cite the two notes of Koppelman [**10**, **11**] and the fundamental papers of Ramírez [**15**] and Henkin [**8**]. Very briefly, Koppelman's papers show the existence of an extensive class of integral formulas, both for functions and for differential forms, of which the Bochner-Martinelli formula is a special case, and opens the possibility of choosing in a rather flexible way a formula appropriate to a given problem. This possibility was exploited by Henkin and by Ramírez, who constructed an integral kernel on strongly pseudoconvex domains. The formula of Henkin and Ramírez is extremely useful for analysis on strongly pseudoconvex domains. In the first place, it is holomorphic: Integration of an arbitrary integrable function against the Henkin-Ramírez kernel yields a holomorphic function; in this it differs from the Bochner-Martinelli formula. Secondly, it is defined explicitly enough that one can use it to obtain good estimates.

The last fifteen years have seen an intense development in several complex variables based on the Henkin-Ramírez integral and its various natural extensions. This work lies particularly in the direction of studying the $\bar{\partial}$-equation, but there are other applications as well.

Let us now turn to the book at hand. It is a faithful translation of the Russian edition, which was published in 1979. In it the reader will find a useful introduction to many of the ideas current in the theory of integral representations together with a good sampling of applications. The reader should approach the book with some knowledge of several complex variables, with a good grasp of Stokes' theorem in a fairly general version, and, as a practical matter, some previous exposure to the rudiments of algebraic topology.

[1] The Russian language literature has devoted more attention to integral formulas. For example, the book of Fuks [3] has a fairly substantial chapter on the subject.

The authors begin with a careful development of the Bochner-Martinelli formula and the general formula of Koppelman. From these they derive, by way of Stokes' theorem, a version of the integral representation formula of Weil for analytic polyhedra, and they construct the Henkin-Ramírez kernel. The general notion of reproducing kernel, from the Hilbert space point of view, is given in a rather summary form. Thus, though the Bergman kernel is introduced in a general setting, it is necessary to look elsewhere for the refined analysis of this kernel that is now available.

Leray's theory of residues is presented in some detail in a chapter that begins with a development of some of the necessary background from algebraic topology. In addition to this essentially global theory, there is some work on the local theory of residues.

The final two chapters of the book are devoted to applications of residues and integral formulas. These applications are mainly to problems in complex analysis, and they amply demonstrate the power of the methods. One of the principal applications is to the study of the local analysis of holomorphic mappings. There is also an application to the determination of certain combinatorial sums. In the final chapter one finds applications of the Bochner-Martinelli formula to the problem of characterizing the boundary values of holomorphic functions on smoothly bounded domains. The Levi problem is solved using integral formulas, which of course, is entirely in the spirit of Oka's original solution of the Levi problem in $\mathbf{C}^2$. Finally, there is a discussion of the application of the Henkin-Ramírez integral to the solution of the $\bar{\partial}$-equation with bounds on strongly pseudoconvex domains.

Thus, the book gives an introduction to some significant directions in modern complex analysis. It is well written, but nevertheless the reader will find certain parts of it rather slow going; integral formulas in several variables are quite complicated. Here and there the reader is sent to other sources for particular results necessary for the development. An attractive feature of the book is a rather large number of nontrivial examples that are worked out in detail and that serve nicely to illustrate the general theory. Several of these will serve to whet the appetite of the reader for a more extended discussion of the applications of integral formulas to complex algebraic geometry. A good source for these developments is [**5**].

Certain topics that one might expect to find here have been omitted. For example, the Poincaré-Lelong formula is not given. An explanation for its absence is, perhaps, that this formula requires as a preliminary a rather lengthy excursus into local analytic geometry to show that varieties have locally finite area. Also absent is a serious discussion of the use of currents in connection with integral formulas, as advocated particularly by R. Harvey. (See, e.g., [**7**].) A final omission we mention is any reference to the very general integral formula given by Gleason [**4**]. This paper is even omitted from the remarkably extensive bibliography the authors have compiled. These omissions do not detract seriously from the book, which makes no claim to being an encyclopedia.

The expert who has followed the Russian-language literature carefully over the last decade and a half will find little that is new here. Other readers, even those who may be quite familiar with the general theory, will find much of interest. In sum, this is an interesting book, which well deserves the attention of those with an interest in the analytic side of several complex variables.

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Angeordnete Strukturen: *Gruppen*, *Körper*, *Projektive Ebenen*, by Sibylla Prieß-Crampe, Ergebnisse der Mathematik und ihrer Grenzgebiete, Vol. 98, Springer-Verlag, Berlin, 1983, ix + 286 pp., $71.00. ISBN 3-5401-1646-X

The study of (totally) ordered fields and groups is a fairly old discipline with venerable roots, the earliest contributions to which go back to the beginning of the century, with results by Hilbert, Hölder, and Hahn. Specifically, Hilbert (1899) considered a special ordered field of real-valued functions in order to establish the independence of certain axioms of geometry, Hölder (1901) showed that every archimedean ordered group can be embedded, as an ordered