applications in approximation theory. A large part of the book is devoted to the following three central problems. In each case the problem can be posed in terms of Tchebycheff systems, generalized Tchebycheff systems, or weak Tchebycheff systems. I. If a GTS is given, does it contain a GTS of order one less? II. If G is a GTS, does there exist a function f such that $G \cup \{f\}$ is a GTS? III. If a function f and an n are given, does there exist a GTS of order n containing f?

For a hint as to how such questions arise, let us cite a theorem of Krein: If $\{1, x, \ldots, x^n, f\}$ is a Tchebycheff system on $[-1, 1]$, then the polynomial p of degree at most n which minimizes $\int_{-1}^{1}|f - p|$ is the polynomial which interpolates to f at the points $\cos k\pi/(n + 2)$, $1 \leq k \leq n + 1$.

The problems mentioned above do not have clear-cut answers in all cases, and work on them continues. Zielke's account of the subject is therefore not final, but it is nevertheless a valuable summary of the current status.

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Applied mathematics: *An intellectual orientation*, by Francis J. Murray, Mathematical Concepts and Methods in Science and Engineering, Volume 12, Plenum Press, New York-London, 1978, xiv + 255 pp.

It is essential from time to time, as the academic world revolves, and as each revolution carries us to new heights of specialization, to refresh our understanding of relationships among disciplines. What has history to do with psychoanalysis, music with computer science, economics with ecology, language with linguistics? It may also be useful on suitable occasions to ask ourselves what a given discipline actually *is* in the contemporary academic context. Professor Murray, Director of Special Research on Numerical Analysis at Duke University, has produced a book that can be regarded as the mark of such an occasion. What, in the rising clamor of academic voices fighting to be heard, *is* Applied Mathematics? Then, having done our best with that, we can examine the relationship forming a central theme of Murray's book. What has mathematics to do with physics? The questions themselves, entirely aside from the character of our answers tend to raise red flags among pure mathematicians. The prospect of finding today's theorem in the design of tomorrow's missile system, or even in next year's solar engines, is discordant with what has become the conventional view of academic mathematics. Here the strongest work is the most abstract and, *a fortiori*, application is evidence of weakness. It may not be unfair to express this view in paraphrase of a remark by Clemenceau: applied mathematics bears the relation to mathematics that military music bears to music.

Readers of the history of mathematics need not be reminded that the growth of support for such attitudes among the majority of our contemporaries–there are, of course, a few virtuoso mathematicians who practice and defend the longer tradition–is recent and swift. To ask for a definition of useful mathematics would have been as puzzling to our academic forebears as a similar question on the nature of science in general. Their answer would be simple and vigorous: the strength of any piece of knowledge is to be found in the step it takes toward our understanding of the world, and thus ultimately to the guidance of our lives. True enough for the natural sciences, we might respond, but have we not matured to the point where the mind of the mathematician works in a world of its own design, carefully quarantined against infection by the motion of atoms and stars? Not possible, say the disciples of Plato and Kant. Your mind is made of atoms and your dream of journeys to the stars is fast becoming real. Try as you may to cast yourself adrift, you still float within the cosmos of which you are the only intelligent part, and your mathematics is neither more nor less than the study of how your intelligence functions.

A thoughtful student of the psychological foundations of mathematics might therefore begin to identify its applications with a set of ideas forming a *gestalt* within the subject itself. From these roots, planted deeply in the human mind, grow the branches that support our only systematic comprehension of nature, and finally the rich harvest of tools and processes that extend our control of the world in ever widening circles. The history of mathematics lends great strength to the metaphor of continuous outward development, just as it confirms the complementary idea of an inward flow of stimuli from natural science back to the mathematical imagination. When we look at the question against this background, we find no clear boundary separating the "pure" discipline from its applications. What we find instead are numerous points along the single tree of knowledge where individuals discover maximum comfort and reward. And the finest mathematicians of the past, as Professor Murray tells us once again, were those of widest range and longest vision both outward and inward.

Physicists take exceptional pride in the fact that more than a few of the most dramatic chapters in the story of mathematics grew from the study of natural phenomena. Discovery of the laws of motion and invention of the calculus was the work of one mind; description of electromagnetism in the form of four beautifully coupled differential equations was the work of another. And two magnificent triumphs in the physics of our own time–general relativity and quantum mechanics–found their natural language in deep mathematical structures. But mingled with our pride in the long and happy marriage between mathematics and physics is an uneasy sense of growing separation and possible divorce. In an authoritative but unpublished essay on the subject, Freeman Dyson offers an explanation and a cure. Interests began to diverge, he says, when Maxwell, who lacked the expository gifts of Newton and his followers, chose to express his ideas in the form heavily burdened with unnecessary mechanical baggage, rather than as a challenge clear and bold enough to attract the attention of working mathematicians. As a result of this misfortune, added to preoccupation with pressing business of their own, mathematicians failed to grasp a rich opportunity. "If they had taken (Maxwell's) equations to heart as Euler took Newton's," writes Dyson, "they would have discovered, among other things, Einstein's theory of special relativity, the theory of continuous groups and their tensor representations, and probably large parts of the theory of partial differential equations and functional analysis . . . simply by exploring to the end the mathematical concepts to which Maxwell's equations naturally lead." (In his famous lecture on the mathematics of special relativity, Hermann Minkowski spoke in the same way, lamenting the fact that mathematicians had not recognized the elegance of the Lorentz group long before it was forced upon them by the physicists.) Following Maxwell, Dyson goes on to say, physicists resorted to a "desperate remedy" to close the growing gap: physics itself split into the "theorists" who understood enough mathematics to describe new phenomena, and the "experimentalists" who labored with equipment in more or less complete ignorance of mathematical theory. The new scheme worked extremely well, bringing spectacular success to the physics of the early twentieth century. During that golden age, until about 1940, theorists like John von Neumann and Hermann Weyl acted as strong bridges between abstract mathematics and laboratory physics, and a handful of highly gifted physicists–Enrico Fermi was the best example–were still able to keep up with everything. But the last forty years have witnessed new and finer splitting among the theorists themselves. One group communicates with mathematicians, the other with experimentalists; they find it increasingly difficult to talk to each other.

Despite the promise of his subtitle, Professor Murray provides neither compass nor map with the help of which his readers might find a reasonable path through the tangle of distractions and conflicts surrounding his subject. Indeed, he has chosen not to recognize the scent of trouble in the air. The denigration of applied mathematics by purists, the loss of continuity from one field to the next, the slipping away of physicists and other students of nature, the atomization of discipline and the disappearance of generalists–these are matters which, although surely of interest and concern to students preparing for careers in mathematics, enjoy no explicit treatment in Murray's book. And yet a hint of their presence will be sensed by perceptive readers who wonder what developments in contemporary mathematics can be compared with magnificent scientific achievements of the past. Murray reviews at length the mathematical treatments of planetary motion, analytical dynamics, manifolds and affine connections, elastic deformation, thermodynamics and probability. He also surveys the histories of arithmetic and geometry; most of his book, in fact, is a *précis* of more or less ancient developments of mathematical tools for physical science. But he has little to say about the climactic events of the twentieth century: after seven pages of mathematics leading to the Riemannian metric, we read only that "this yielded a more precise description of gravitation in the solar system;" the paragraph or two mentioning quantum mechanics exhibit no mathematics; recent results in general relativity, catastrophe theory, linear programming, econometrics, mathematical biology and information theory get no space at all.

The only example of modern analysis Murray chooses to treat in detail–it occupies a quarter of the first substantive chapter–is a study of motion of aircraft in flight, its aim being "to illustrate the ideas associated with technical simulations, in particular, block diagram, math model, flow chart, and scenario . . . " The analysis appears to have been published in a technical report to the United States Naval Training Center in Orlando. The rigid-body dynamical theory for such a problem is entirely Newtonian and the mathematical level would have been well within the grasp of a nineteenth-century physicist. What gives the exercise contemporary flavor, of course, is its application to the design of flight simulators for pilots in training. But this surprising choice from what must certainly be a rich stock of current problems can be most easily understood as an accomodation to Murray's conception of useful research. "Applied mathematics," he writes, "is usually part of a large effort under contract with the Federal Government and based on scientific and technical understanding. It is a team effort and documentation is essential." The same theme pervades the introductory chapter, where we find three exhaustive tables of data on government expenditures for Research and Development over the years. Such things are doubtless interesting and important to students planning careers in Federally supported research, but it is nonetheless astonishing to find budget figures and the "documentation" of a Government contract on the opening pages of an "intellectual orientation" for readers who are likely to be inclined differently–tempted, perhaps, to emulate giants of the past whose accomplishments occupy the rest of the book.

Both in the design and in the execution of this work, Professor Murray fails to produce the eloquent defense of useful mathematics that we hope somewhere to find. Far from attracting gifted students to a noble profession by making a strong case for broadening and intensifying the influence of mathematics on human affairs, this book–especially in its opening chapters–can only strengthen the position of the purists. Examined more narrowly as a textbook of history and technique, the book reveals other smaller flaws. Its organization is haphazard, as if the manuscript were stapled together from short pieces produced for a variety of different purposes. In some cases there is little connection between what Murray says he is doing and what he actually does. Why should the study of invariant tensors be a part of *Natural Philosophy* while the kinematics of rigid bodies falls partly into the chapter on *Simulations* and partly into *Energy*? The summary of a section on group theory promises a treatment of its significance in modern physics. We expect, of course, a reference to Wigner's contributions to quantum mechanics or to recent work in the theory of elementary particles. We read instead that the mechanics of special relativity, governed by the Lorentz group, gives an exact description of the motion of the planet Mercury–which is in fact not true: the problem was solved in the framework of general relativity. The style of Murray's writing is itself a threat to clarity. Here, from a section entitled "*Intellectual ramifications*, is a specimen far from untypical: "Problem mathematics is necessarily associated with the idea of human affairs based on a mutual understanding arrived at by logical means." Finally, the exercises at the ends of chapters of this book deserve notice. Some are mysterious ("List the propositions in Euclid that are incorrect;") some are incomplete ("An airplane makes a tight turn so as to 'pull 8 g's.' What was the minimum radius of curvature?") some are meaningless ("Show that a mole of one perfect gas differs from that of another perfect gas only in density;") some go beyond the point of reasonable demand ("What problems were considered by Archimedes? Cavalieri? Wallis? Newton? Euler? Laplace? Gauss? Cauchy? Riemann? Weierstrass? Cantor? Borel? Lebesgue? Hilbert? L. Schwartz?") and some are simply impossible ("There are two major types of nuclear explosive devices. Describe the mathematical formulation of the action in each case.")

At the close of his provocative essay on the gap between disciplines, Freeman Dyson writes of ways to bring mathematics and physics back together. It is at present not realistic, he says, to expect members of one group to make original contributions to work of current interest in the other. The fields have drifted too far apart and their union is too large for a single intelligence to span. What can be done, at least for the time being, is to establish contact through papers of a special kind: when a new result in one field shows promise of attracting interest in the other, a review article gathers up points of contact and proposes areas of collaboration. It will, of course, take a change of attitude, the invention of new kinds of reward, and a few reforms in graduate education to make this happen. It is perhaps just barely possible. And what about a more intimate reconciliation of the sciences in the long future? Here our imaginations must range more freely. The solution–a dangerous one, says Dyson–lies in the hands of the biologists who will ultimately discover ways of extending human memory and intelligence to the point where the whole of science is once again comprehensible to one human being. Meanwhile we must do what we can with the natural mind as it is given to us.

An excellent book on the present status and possible future of useful mathematics would be a step in the direction that Dyson envisions. Professor Murray's first attempt falls short of the requirement. Perhaps he, or another mathematician of equal distinction and equal dedication to the task, will give it another try.

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Relativistic theories of materials, by Aldo Bressan, Springer Tracts in Natural Philosophy, vol. 29, Springer-Verlag, Berlin-Heidelberg-New York, 1978, xiv + 290 pp.

Einstein's general relativity is primarily a unification of gravity with spacetime geometry: the curvature of a four-dimensional Lorentzian manifold signals the presence of gravity. But the theory can be regarded as a complete description of at least macrophysics; it necessarily deals with electromagnetism and matter in addition to gravity. In fact its most important specific postulate, the Einstein field equation $G = T$, describes, roughly speaking, how matter and electromagnetism generate gravity. The equation relates a purely geometric object with a physical, almost anti-geometric one: G, the Einstein curvature, is determined at a spacetime point by certain averages of the sectional curvatures there; T, the stress-energy tensor field, is determined by electromagnetism and matter. Einstein's own attitude toward this contrast is given, for example, by his comments on the equation in his autobiographical contribution to *Albert Einstein, Philosopher-Scientist* (Paul A. Schlipp