Authors' and lecturers' insensitivity to the needs of most readers and listeners for basic down-to-earth examples is one of the greatest sins mathematicians commit and only tends to reinforce barriers between disciplines. For example, it is now evident that the subjects of bifurcation theory and singularities of mappings have a great deal in common and a great deal to offer each other; a brick wall of noncommunication and reticence took almost a decade to break down. (Nirenberg's Courant Institute notes on Nonlinear Analysis were instrumental in this bridging.) There *is* room for improvement in Dieudonné's treatise in this respect.

6. Overview. Most mathematicians write advanced books for themselves; to set down their views for the record, to educate a close circle of followers or simply for their own ego, prestige or promotion. It is a rare mathematician who is earnest about making the necessary effort to break down barriers and to further mathematical evolution by teaching aspiring mathematicians with sensitivity and understanding. I believe Dieudonné is one of this rare breed.

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Infinite dimensional linear systems theory, by Ruth F. Curtain and Anthony J. Pritchard, Lecture Notes in Control and Information Sciences, vol. 8, Springer-Verlag, Berlin-Heidelberg-New York, 1978, viii + 297 pp., $14.80.

As I have been interested in this area for a number of years, it may be significant to say at the outset that this is a book I wish I had written myself. The book literature in infinite dimensional linear systems, or distributed parameter systems, to use the favored engineering terminology, is not very extensive. Perhaps the first significant contribution to that select category was A. G. Butkovsky's *Theory of optimal control of distributed parameter systems* (American Elsevier Publ. Co., New York, 1969; the Russian version appeared in 1965). This pioneering work was followed by J. L. Lions' landmark volume *Optimal control of systems governed by partial differential equations* (Springer-Verlag, New York, 1971; the French version appeared in 1968). There have, of course, been numerous journal articles, published conference proceedings, bibliography listings and review articles. In one of the last (SIAM Review, Vol. 20, 1978) I took some pains to point out that there is a distinct difference between the notions and methodology of *optimal* control and those of general control systems theory. The latter is a study of dynamical systems which involve control parameters explicitly intended for use in modifying systems behavior together, usually, with a set of admissible observation functionals providing information on the system state. Systems theory primarily involves such concepts as controllability, stabilizability, observability, etc. While the Butkovsky and Lions contributions do not entirely neglect these basic systems theory concerns, I think it is fair to say in both cases that the emphasis was placed on optimization. Thus it is particularly gratifying to be able to welcome the work by Curtain and Pritchard which, while not neglecting optimization by any means, has the basic systems theory of infinite dimensional linear systems as its core.

The challenge facing the mathematician who works in infinite dimensional linear systems theory is, to a large extent, that of taking the highly unified and complete theory which exists for finite dimensional linear systems and extending it, insofar as possible, to the infinite dimensional case. Of course, in carrying this out he does encounter genuinely new phenomena with no finite dimensional counterpart. Resorting to a military metaphor, one can choose to attack on a wide front, capturing at best a few "trenches" or one can initiate deep piercing sorties at what appear to be vulnerable points in the line. Greater penetration is ordinarily achieved with the second strategy but one is left with the problem of creating some kind of unified front out of the collection of discrete breakthroughs. In the control theory of infinite dimensional systems a great deal is known about certain specific systems–the ones familiar to any graduate student in applied mathematics, such as the wave equation, the heat, or diffusion, equation, neutral and retarded functional equations, and so on. Each of these types of systems will, to cite one approach, be related via resolvent operators, Laplace or Fourier transforms, etc., to certain classes of entire functions which have been exhaustively studied by famous mathematicians for many years. The very detailed properties of these function classes can be brought to bear on questions of controllability, stabilizability, placement of spectra via feedback, and other systems theoretic properties. When it comes to the development of a systems theory applicable to infinite dimensional systems in some generality, say for the whole class of systems associated with strongly continuous semigroups in Hilbert or Banach spaces with general additive forcing terms, one resorts to the body of relevant theory in functional analysis. Some very fine results have been obtained, many of them set forth in the volume under review here but, understandably, one does not obtain a lot in the way of fine detail by this approach.

The dilemma, in writing a book of this sort, is to decide which of the above approaches to use, or alternatively, to decide on an appropriate balance in a study which makes use of both strategies. Curtain and Pritchard have followed this second route and the balance achieved is a very nice one indeed. A very wide collection of important concrete systems are introduced at various points along the way but the basic theorems are obtained in a rather broad setting.

Having indicated my general approbation, I will pass now to some remarks, unfortunately superficial, on the book's contents; such criticisms as I may voice should not be viewed as detracting from my overall enthusiasm for a difficult job very well done.

This is not the sort of book that the nonspecialist can expect to pick up and assimilate between invidious distinctions at the next department meeting. True, the authors do make an attempt to review finite dimensional systems theory but the effort is astonishing for its brevity more than anything else. Probably this is all to the good. The nature of the discipline is such that it cannot really be appreciated without enough background to understand why certain processes are important and why a given theorem, on the face of it rather tedious, is really quite significant.

Control theory, in our way of looking at it, represents a new departure of sorts in applied mathematics. Classical applied mathematics emphasizes natural phenomena and seeks to express, in mathematical terminology, the laws governing the evolution, or perhaps describing a steady state, of a natural system. But the system stands out there more or less *in vacuo*, having little interplay with the rest of the cosmos. In control theory we suppose that the system "has some handles on it" whereby a "system operator" (human, machine, or composite) is able to influence the way in which the system behaves. In practice this external authority is often delegated to an automatic mechanism once a control law (a rule for specifying what control should be exercised when the system exhibits a given state) has been formulated mathematically. An example is that of an aircraft autopilot which performs the routine chores of keeping an airplane in trim and on course, and perhaps improving stability characteristics during difficult maneuvers. The human crew can perform these tasks, and do so at times, but the automatic device saves a great deal of meaningless fatigue and allows time for a cup of coffee and a wink at the stewardess.

Infinite dimensional systems come into the theory in the same way as in applied mathematics generally. What is involved is a certain distributed character of the mode of description of system states. An example is the temperature of a long strip of metal fed through a rolling mill. The temperature is a function of position on the strip, displaying a continuous profile of values. If this profile is uneven, the rolled steel strip will also be correspondingly uneven. Another example, much different in origin but very similar from the mathematical point of view, is the evolution of the demographic profile of a nation's population, where, as we all know, baby booms and baby busts (?!) are reflected years later in high demand and fat salaries for academics in the first instance and exile to South Central State Teacher's College and four classes per day (if you're lucky) in the second. Each of these processes can be controlled to some degree, the first much more than the second.

Of course one has to have some tools at hand to treat with infinite dimensional systems. The authors provide a review of semigroups of operators and related material, including the basic Hille-Yoshida existence-uniqueness theory. The "straight line" reader will, assuming only incidental interest in this subject *per se*, find tough slugging here. From the didactic point of view I would be fearful of losing much of my audience at this point in Chapter 2. I would advise the general reader to skim this chapter, go on to Chapter 3 and return to Chapter 2 for fresh mathematical provender as necessity indicates and patience permits.

Chapter 3 is right where it ought to be, a thorough treatment of controllability, observability and stabilizability theory (more on the second of these shortly). I like the form given to the conrollability and observability theories as well as the duality theorem but I again feel that too many abstract theorems are proved before the algebraic structure is adequately related to the reader's (assumed) control interests. Also, it seems to me that the authors missed a fine opportunity to emphasize that a very wide variety of control operators, and related notions of controllability, exist in addition to those of null controllability (control to an equilibrium point in practice), arbitrary final state controllability and the others which they list. I feel that the treatment of stabilizability (taking a system with nonexistent or inadequate stability properties and, by applying a control determined via linear state feedback, making the system asymptotically stable or improving any stability characteristics which it already possesses) to be rather minimal here. This is unfortunate because, in practice, this is the most important mode of control which is used. The references extend further but are not commented upon in the text.

The exercise of control has an ineluctable corollary: the necessity of observation in order to monitor the state of the process. This dependence of effective control on observation is one that totalitarian governments, e.g., have been among the first to appreciate. But it does not follow that a practitioner of control theory is philosophically aligned with Torquemada or Beria. Indeed, some of the most elegant work in the entire subject concerns the effective use of minimal information; minimal both in quantity and quality. Typically the control specialist contents himself, willingly or otherwise, with measurement of only a few system components. Such measurements as are permitted are still indirect, in that the measuring devices are dynamical systems in their own right so that the reading obtained is some sort of transform of the quantity being monitored, and are noisy, to a greater or lesser degree, due to disturbances impinging upon the system or the measuring device from a multitude of sources. The problems of dealing with information limited in quantity and quality are dealt with in observer theory and filtering (estimation) theory, respectively.

The second of these topics is very nicely treated in this book–it might be viewed as the *pièce de résistance* of the fare served up by the volume as a whole. The first, insofar as the main question of observer theory is whether or not a given collection of measurements is adequate for purposes of control or stabilization, is hardly mentioned. This is forgivable since observer theory remains one of the more arcane aspects of the finite dimensional theory and has been minimally developed, if at all, in the infinite dimensional context. Chapters 4 through 7 extend the linear quadratic optimal control theory and linear Gaussian filtering results to infinite dimensional systems. The presentations are praiseworthy and I particularly appreciate the detailed examples presented in connection with the linear quadratic optimal control theory as it applies to certain diffusion systems and delay equations. The introductory treatment of stochastic processes is succinct but thorough–the reader had best come to play ball, mathematically speaking. In Chapter 6 I am happy to see smoothing and prediction treated as well as state estimation. Chapter 7 treats the "separation theorem" for optimal control with restricted observation. This is the remarkable and, at least to me, nonintuitive result that the problem of optimally controlling a noisy system using information which is less than the full system state can be decomposed into the separate problems of optimally controlling the system with full state information and estimating the full system state on the basis of the available measurements. The treatment here seems unusually readable and has the additional attraction, which devotees of the subject have learned not to take for granted, that the separation theorem is correctly stated.

The last two chapters treat subjects which are "nonstandard" in one sense or another. In Chapter 8 the authors discuss unbounded control input elements and sensing functionals. This subject has no finite dimensional counterpart but is "of the essence" in discussing control and observation of processes described by partial differential equations where control variables appearing in the boundary conditions and state measurements made at boundary points are important both because they are physically the most realizable and because, mathematically, they tend to provide the strongest controllability and observability results. The treatment is admirably general from the abstract point of view but appears, at first reading to be limited in application to analytic semigroups. This is certainly forgivable since the corresponding theory presented in a context wide enough to include, e.g. hyperbolic systems, would necessarily be very complicated and would have to resort to description of a number of special cases. Chapter 9 is a discussion of time dependent infinite dimensional systems carried out in the context of quasievolution operators and generators This material is nonstandard to the degree that in the differential equations and functional analysis literature generally, the extension to infinite dimensional systems of the very complete theory of finite dimensional linear systems with time varying coefficients is fraught with all sorts of technical difficulties. The main objective of this section is the study of the linear-quadratic optimal control problem (linear system, quadratic objective function) for a time-varying infinite dimensional system on a finite time interval.

The book has ample lists of references, one following each chapter and a long supplementary list at the end. I do feel that more commentary on the nature of these referenced contributions in the body of the text would have been helpful but this is mere quibbling. The book is a very sound one and the authors are to be congratulated on a significant achievement.

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Constructive functional analysis, by D. S. Bridges, Research Notes in Mathematics, Volume 28, Pitman, London-San Francisco-Melbourne, 1979, vi + 203 pp., $15.00.

The appearance in 1967 of Errett Bishop's book *Foundations of constructive analysis* was a significant event. Until then it had been a commonplace that the constructive point of view toward mathematical truth could be successful only in a few areas of mathematics, and certainly not in analysis or topology. That is, it was believed that these areas would inevitably be trivial and uninteresting if constructive principles were followed. Errett Bishop deflated this opinion by showing explicitly how to develop a substantial portion of abstract analysis constructively. Moreover, the subject matter and style of