for these classes of functions. We hope that future volumes might consider these matters.

Integral representations will prove a valuable reference for experts. By its very technical nature, it contains no results with the compelling elegance of, say, the Riemann mapping theorem. But the techniques are far more important than the specific results and, by that measure, the book is a success.

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The algebra of random variables, by M. D. Springer, Wiley, New York, 1979, xxii + 470 pp., $26.95.

It is both surprising and regrettable that it took over 30 years after the appearance of the pioneering paper by B. Epstein in (1948), *Some applications of the Mellin transform in statistics*, to produce the first text on the *Algebra of random variables* which is based on an elaboration and extension of Epstein's ideas. It is also equally unfortunate and puzzling that Epstein's paper appeared so late in this century, over ten years after Cramér's classical *Random variables and probability distributions*. The book under review is indeed very close in its mathematical content to the treatises on the subject matter of the special functions which originated in the beginning of this century. In fact, the mathematics in this book could fit very well into Whittaker and Watson's *Modern analysis* (1915).

This lag of about half a century is–in my opinion–due to two basic reasons: the awkward and uncertain position of probability theory (and thus indirectly what is known today as "statistical distribution theory") within the framework of all the mathematical disciplines which lasted at least until the publication of Kolmogorov's axiomatization in 1933 and to some extent to a certain contempt exercised by the editors of some mathematical and statistical journals in the fifties and early sixties towards problems of statistical distribution theory which were dismissed (and not without justification) as exercises in advanced calculus. These "exercises" were for a long time considered to be unpublishable in reputable journals under the mistaken assumption that any graduate student can carry out these "straightforward" calculations without much difficulty. On the other hand, these problems were far too demanding for investigators in various branches of science and engineering who encountered them in the course of their applied research. It is only in the last decade or two that we have witnessed a change in the attitude towards "routine" calculations which was prompted to a great extent by the penetration of computer-aided methods in mathematics.

Professor Springer performed an important service to applied mathematics by collecting in a single volume results dealing with derivations of sums, differences, products and quotients of random variables and by pointing out the importance of these results in statistical and engineering applications. He studied over 300 references scattered in periodical literature (including numerous obscure journals) and presented many of them in a unified and admirably lucid and well organized form.

In Chapters 3, 4 and 5, distributions of sum and differences, distributions of products and quotients and distributions of algebraic functions *respectively* are discussed. (The omission of Aroian's tables of products of two normal random variables is somewhat regrettable.)

The first very short chapter contains a historical Introduction while Chapter 2 (of about 50 pages) is devoted to those elements of complex variables that form the basis for integral transform methods that is one of the major mathematical tools utilized in the book.

One may perhaps quarrel with the author's conviction that the material in this chapter is sufficient for an uninitiated reader to comfortably follow and verify the rather elaborate calculations presented in the starred chapters of and Appendices to the book; however, the information is presented with ample motivation.

(A number of misprints in Chapter 2 were pointed out to the reviewer by R. M. Norton. Among those: in equation (2.8.6b) on p. 28, $f(x)\,dx$ on the right-hand side should be replaced by $F_t(f(x))\,dt$ and inconsistencies in Section 2.8.2 (pp. 34–36) which are due to the fact that the Fourier transform is defined therein with a "+" in the exponent whereas properties such as $F_t(x^n f(x)) = i^n F_t^{(n)}(f(x))$ are valid for a "−" definition.)

The sixth and seventh chapters are, in the reviewer's opinion, the most innovative and mathematically interesting (and demanding) part of the book. Here distributions of algebraic functions of independent H-function variables and evaluation of the H-function inversion integrals respectively, is discussed. Most of the material presented in these two chapters are based on the original research of the author and his ex-students, B. D. Carter and J. N. Lovett. (Carter's work appeared in the SIAM Journal of Applied Mathematics in 1977, based on his 1970 Ph.D. thesis. Lovett's reference is to his Ph.D. thesis of 1977.) The H-function distribution employed by the author is a positive real-valued variant of Mellin-Barnes' integral (see, e.g., Erdelyi, Vol. **1**, pp. 49–50). The latter encompasses a great variety of classical special functions including Gauss' hypergeometric function, the confluent hypergeometric function, Meijer's G-function and the Bessel function, to mention only a few of the most well-known particular cases. Correspondingly, the H-function distribution contains as special cases numerous continuous distributions such as Gamma, Weibull, Maxwell, Beta, half-normal, Chi-squared, F, and general hypergeometric distributions. The main attraction of the class of these distribution functions is that it is closed under the operations of the product, rational powering and taking a quotient. (It is *not* closed under addition.) Moreover, the author's associates developed analytic models for evaluating the H-function inversion integral using standard techniques of complex integration involving Jordan's lemma and Bromwich paths which are presented with painstaking detail in Chapter 7. This is followed by 2 examples of a rather simple nature, one deriving the product of two Weibull variables and the other calculating the product of two Beta variables. The reviewer would have hoped for somewhat more elaborate examples which would justify more convincingly the applicability and usefulness of the rather lengthy but basically straightforward method developed in this chapter. Nevertheless, coupled with Carter's computer program described in Chapter 8–devoted to approximating methods–which permits us to evaluate moments of algebraic functions of H-function random variables and the use of these moments to approximate the resulting distribution function, the theory seems to apply in much more complex situations. The author advocates T. W. Hill's approximating procedures which are reproduced in the book, originally developed in Hill's doctoral dissertation of 1969 (although other methods are available at present). The author then discusses various methods of approximating continuous distributions using such well-known techniques as the Gram-Charlier series, Laguerre polynomial series, step function approximation and others and evaluate the accuracy based on Fourier Sines series, including examples of error evaluation. Again the examples are not as convincing as one might have hoped considering the claimed generality of the methodology.

The final Chapter 9–dealing with the distribution problem in statistics again purports to be a demonstration (and a rather convincing one) of the applicability and advantages of the integral transforms and convolutions for deriving various sampling distributions ranging from the arithmetic, geometric and harmonic means, the "classical triple" of t, χ^2 and F, some noncentral distributions, up to distributions of products and quotients of order statistics from various continuous distributions and sums, differences, products, quotients and linear functions of Bessel random variables.

One may object to the author's assertion concerning the blanket superiority of the integral transforms method, since some of the results may be obtained mathematically more elegantly and insightfully using moment generating functions and especially using geometric considerations (as the author himself hints in the introduction). However, from the practical and purely computational aspects, the methodology recommended by the author does provide a sound and generally unfailing route.

One may also question the general philosophy of the book based on an almost total submission to the techniques rooted in the theory of integrals of functions of a complex variable for solving distributional problems. In fact, there are some recent indications that these results can be obtained using less artificial and more direct methods (D. N. Shanbhag, 1979).

In spite of these reservations there is no doubt that the book has very successfully filled a longstanding gap in the literature and will be of immense usefulness to applied probabilists, statisticians and qualitatively oriented researchers dealing with probabilistic modelling who up until now were deprived of a comprehensive and carefully written compendium on elementary algebraic operations with one-dimensional random variables. It is hoped that a future edition of this book will incorporate a more detailed discussion of the algebra of multi-dimensional-random variables in view of the substantial demand and increase in applied probabilistic models based on multivariate distributions.

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Unitary group representations in physics, probability, and number theory, by George Mackey, Benjamin/Cummings, Reading, Mass., 1978, xiv + 402 pp., $19.50.

Even to one who does not wish to "buy" group representations as the end all or be all that it sometimes pretends to be, this is a very nice book. In particular, with Jauch's *Foundations of quantum mechanics* [1] on the one side and the present *Unitary group representations* by Mackey on the other, one has some forceful and interesting arm-chair reading in store. One should also keep Dirac's *Principles of quantum mechanics* [2] close at hand. In fact this reviewer was struck by and reminded of Dirac's elegant style and spare exposition when reading the present account by Mackey, even though the formats are different.

Mackey's book is the now published version of what are commonly called Mackey's Oxford notes [3] for lectures given there in 1966–1967. Although the treatment does include some discussion of applications to or perhaps more correctly relations to topics in probability, number theory, statistical