Further, Kleinberg does not mention the problem of computing δ_n^1, a problem which has motivated a great deal of the theory to which these notes contribute. As a result of these omissions, the nonexpert who does not simply love the bizarre will find little reason to read past the introduction.

How should one use the book? Curiously, although its natural context is a quite elaborate and sophisticated theory, only in Chapter 2 does the book require more than the basics of set theory. Nevertheless, the nonexpert who wants to learn something of the set theory of $L(\mathbf{R})$ assuming $\mathrm{AD}^{L(\mathbf{R})}$ would be much better advised to start elsewhere, perhaps by reading [2], [3], [4] (in that order). He will be rewarded with a broader view of this field. On the other hand, the expert in the field will find these notes a useful and well-written reference on one of its aspects.

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JOHN STEEL

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Several complex variables, by H. Grauert and K. Fritzsche, Graduate Texts in Math., vol. 38, Springer-Verlag, New York, Heidelberg, Berlin, 1976, viii + 206 pp.,$18.80.

There is no shortage of introductory texts and treatises in several complex variables. Since the subject borrows methods from diverse areas of mathematics such as algebraic geometry, functional analysis, p.d.e., differential geometry and topology, and thus partially overlaps with these areas, it is not surprising that these introductory books cover a wide variety of material from various perspectives. [A listing of some of the recent introductory texts (in English) in several complex variables is given in the references below.] Grauert and Fritzsche's *Several complex variables* is a more or less orthodox introduction to the classical themes of several complex variables arising out of the Oka-Cartan theory. The classical global theory of functions of several complex variables is based on the existence, noted by Hartogs, of domains D in $\mathbf{C}^n$ ($n > 1$) such that every holomorphic function on D can be extended to a larger domain. If on the contrary D is the natural domain of definition of a holomorphic function, then D is called a *domain of holomorphy*. The subject received its major impetus from attempts to describe domains of holomorphy. The Cartan-Thullen Theorem characterizes domains of holomorphy as domains D that are *holomorphically convex*; that is, the hull of any compact subset with respect to the algebra of holomorphic functions on D is compact.

Later, Oka, together with Norguet and Bremermann, gave another characterization of domains of holomorphy as domains which carry a $\mathcal{C}^2$ function p such that $(\partial^2 p/\partial z_i \partial \bar{z}_j)$ is positive definite and the sets $\{p \leqslant r\}$ are compact for all r (*pseudoconvexity*). H. Cartan, drawing upon the work of Oka combining the global and local theories, gave a proof that $H^q(D, \mathfrak{S}) = 0$ for $q > 0$ if D is a domain of holomorphy (or more generally a Stein space) and $\mathfrak{S}$ is any coherent analytic sheaf. This vanishing theorem, known as "Cartan's Theorem B", has numerous uses. For example, it gives a solution to the "Cousin problem" on the existence of global meromorphic functions with prescribed poles on a domain of holomorphy.

The book by Grauert and Fritzsche is more elementary than most texts in several complex variables and would be suitable for a one-semester course for beginning graduate students. A strong point of the book is its excellent, carefully executed illustrations and its numerous examples. The authors state without proof many of the deeper results of the subject in order that the novice can sense the direction of the subject before struggling through the lengthy proofs of the harder theorems. Not many texts in any mathematical subject provide this kind of preview, and the authors' departure from the traditional definition-theorem-proof approach is quite welcome. On the negative side, the first chapter contains some slightly obscure definitions and poor notation, and although the remainder of the book is quite clearly written, a beginning student may nonetheless have difficulty understanding the first chapter. Also, many of the proofs in the book are obscured by too much elementary detail.

Of the similar introductory texts in several complex variables, Bers [**1**] and Gunning and Rossi [**4**] are more advanced and are suited for a year's course. Both of these other texts include proofs of "Cartan's Theorems A and B" and the Grauert-Narasimhan proof of Oka's theorem on the Levi problem. These major theorems are stated but not proved in Grauert and Fritzsche. Both Bers [**1**] and Grauert and Fritzsche do not discuss much local theory beyond the Weierstrass preparation theorem, although Grauert and Fritzsche gives the local description of analytic hypersurfaces. Of these books, Grauert and Fritzsche contains the best treatment of the general theory of sheaves and cohomology, and it is unfortunate that this treatment is not followed by a proof of Theorems A and B. Bers [**1**] has the usual disadvantages of a lecture-notes format: It is mimeographed and contains no illustrations, index, or bibliography. Gunning and Rossi [**4**] contains several errors. On the other hand, Grauert and Fritzsche is carefully prepared with ample illustrations and few misprints.

I shall now briefly describe the contents of the book. The first chapter introduces the concept of a holomorphic function and discusses Reinhardt domains, the Cauchy Integral Formula in $\mathbf{C}^n$, and the Cauchy-Riemann equations. Complex differentiability is defined in a slightly obscure way, and the reader is not told until four pages later that this concept is equivalent to the function being holomorphic (having convergent power series expansions). The authors introduce the standard multi-index notation for power series, but do not fully use it and instead give some proofs in $\mathbf{C}^2$. The book contains an excellent introduction to the classical global theory, including descriptions of

Hartogs domains, holomorphic convexity and Levi pseudoconvexity. Proofs of the Kontinuitätssatz, the Cartan-Thullen theorem, and the existence of the envelope of holomorphy of a Riemann domain are given. (The simple geometric condition for a Reinhardt domain to be a domain of holomorphy is not given, the authors thus missing an opportunity to provide a large class of examples.) There is a brief introduction to the local theory. The Weierstrass preparation theorem is proven using normed power series. A good local description of analytic hypersurfaces is provided, but the local parametrization of subvarieties of arbitrary codimension is not given, and the Nullstellensatz is not even stated. Facts about dimension are stated without proof.

The general theory of sheaves (emphasizing the étale-space approach) and cohomology, including coherence and Leray's theorem, is provided. The treatment of sheaves is excellent. However, sheaves of abelian groups are not defined, although they are used later on, and not enough examples of sheaves are given. The coherence theorems of Oka and of Cartan and Theorems A and B are stated without proof, and applications are given. The book concludes by describing the exterior algebra of (p, q)-forms on complex manifolds and proving the Dolbeault lemma and Dolbeault theorem. (The authors use $i \cdot$ instead of J to represent the almost-complex structure, so that $i \cdot \partial/\partial x_\nu = \partial/\partial y_\nu$, which might confuse the reader.)

The book is an English translation of the original German version [*Einführung in die Funktionentheorie mehrerer Veranderlicher*, Springer-Verlag, 1974]. The translation is excellent throughout; my only criticism of the translation is the use of the term "topological map" instead of the more common "continuous map".

Several complex variables by Grauert and Fritzsche is overall a good elementary introduction to this subject. The book emphasizes global theory and provides excellent discussions with detailed proofs of the classical global theory–the Cartan-Thullen theorem and holomorphic hulls–and the modern theory of sheaves and cohomology. In order to remain elementary it does not prove Theorems A and B. A shortcoming of the book is its limited discussion of the local theory.

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BERNARD SHIFFMAN

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Algebraic methods in the global theory of complex spaces, by Constantin Bănică and Octavian Stănăsilă, J. Wiley, London, New York, Sydney, Toronto, 1976, 296 pp., $21.50. Revised English version of original Romanian text published in 1974.

The algebraic methods referred to in the title are primarily those of sheaves and cohomology. This book presents an account, with complete proofs, of recent advances in the theory of complex analytic spaces.

Complex analytic spaces are complex manifolds with singularities based on the local model of zeros of a finite number of holomorphic functions in an open set in $\mathbf{C}^n$ (an analytic variety). A sheaf over a space is essentially an assignment to each point of the space of local data near that point (e.g. holomorphic functions near a point, holomorphic functions which vanish on a given subvariety, etc.). Sheaf cohomology is an obstruction theory which encodes information on passing from the solution of certain local problems to certain global problems. For instance, a subvariety is defined as a closed set which is locally the zeros of holomorphic functions, and one can ask whether this same subvariety is the zero set of globally defined holomorphic functions. The use of sheaves and cohomology in the study of complex spaces has been extremely fruitful since the initial work of Cartan and Serre in this direction in the 1950s.

The statements of results in this book are closely parallel to the analogous results in algebraic geometry, mostly due to Grothendieck, though the proofs are different, and usually more difficult than in the algebraic case. This approach should appeal to algebraic geometers wishing to learn about complex analytic spaces, since it highlights the similarities between the two theories. It should also be useful to specialists in several complex variables, since it assembles very recent and widely scattered material. There is no index, but there is an extensive bibliography, in which 3/4 of the articles have appeared since 1960.

The idea of using sheaves and cohomology in the study of several complex variables is not new. Sheaves were introduced by Leray (1950) to study the cohomology of fibre spaces. One of their first serious applications was by H. Cartan (with the assistance of Serre), in his Paris seminars of 1951/1952 and 1953/1954, where he used them to reinterpret and expand the work of Oka, thus laying new foundations for the theory of several complex variables. Grauert and Remmert built upon this foundation with a series of papers, including the development of the concept of a complex analytic space.