become the standard reference for young workers in and students of algebraic geometry. They will be well served. Because algebraic geometry is important for so many fields from partial differential equations through complex analysis to number theory and algebra, this book belongs on every mathematician's shelf. We owe Hartshorne our thanks.

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Infinitary combinatorics and the axiom of determinateness, by Eugene M. Kleinberg, Lecture Notes in Math., vol. 612, Springer-Verlag, Berlin, Heidelberg, New York, 1977, 150 pp.,$8.30.

Many questions of mathematical interest cannot be answered on the basis of ZFC, the standard axiomatization of set theory. Notable examples are the Continuum Problem, and the problem of the Lebesgue measurability of PCA sets of reals. (PCA sets are the projections of complements of analytic subsets of $\mathbf{R}^2$.) However, as Gödel suggested in [1], it may be possible to settle such problems by extending ZFC. Gödel hoped to find new axioms with the same "intrinsic necessity" as those of ZFC. Failing this, he hoped that "there might exist axioms so abundant in their verifiable consequences, shedding so much light upon a whole field, ... that, no matter whether or not they are intrinsically necessary, they would have to be accepted at least in the same sense as any well-established physical theory." One might call the search for and study of such axioms "Gödel's programme"; it is the antithesis of Hilbert's programme.

Work on this program has concentrated on two sorts of hypotheses, the first sort asserting the existence of certain large cardinal numbers, and the second the determinateness of certain definable games. Both sorts can be viewed as extrapolations of principles inherent in ZFC, though neither has anything like the "intrinsic necessity" of ZFC. (Some surprising equivalences between hypotheses in these two very different families have been discovered, which suggests that these two ways of extending ZFC are not only compatible but ultimately the same.) Neither sort settles problems which, like the Continuum Problem, involve the notion of an arbitrary set of reals, but both sorts have had success with problems about definable sets of reals. Determinateness hypotheses have been much more fruitful in this respect.

By "game" we mean an infinite, two person game in which the players alternate playing natural numbers, producing at the end as a "run" of the game an element of the Baire space ${}^{\omega}\omega$. A game is Γ-definable if its payoff set, the set of winning runs for the player moving first, is Γ-definable. A game is determined if one of the players has a winning strategy. One can view much of classical descriptive set theory as based on the principle that open (i.e. $\boldsymbol{\Sigma}^0_1$-definable) games are determined. The determinateness of open games and in fact of Borel ($\boldsymbol{\Delta}^1_1$-definable) games is provable in ZFC. The determinateness of analytic ($\boldsymbol{\Sigma}^1_1$-definable) games (which is equivalent to a large cardinal hypothesis) already represents a significant extension of the classical theory beyond ZFC; for example, it implies the Lebesgue measurability of PCA sets of reals.

The notes under review are best seen as based on the hypothesis $\mathrm{AD}^{L(\mathbf{R})}$ that all games in $L(\mathbf{R})$ are determined, where $L(\mathbf{R})$ is the universe of sets constructible from the real numbers as urelements. $\mathrm{AD}^{L(\mathbf{R})}$ is a strong assumption indeed; strong enough to settle the classical questions about sets of reals in $L(\mathbf{R})$ which cannot be settled in ZFC, and seemingly stronger than any known large cardinal hypothesis. Now ZFC + $\mathrm{AD}^{L(\mathbf{R})}$ implies that $L(\mathbf{R})$ is a model of ZF + AD + DC, where AD is the assertion that all games whatsoever are determined, and DC is the axiom of dependent choice. Thus in studying $L(\mathbf{R})$ under the hypothesis $\mathrm{AD}^{L(\mathbf{R})}$ one often reasons in the theory ZF + AD + DC. This is what Kleinberg does. Note that AD is false in the full universe V of sets, since using a well order of the reals one easily constructs a nondetermined game.

Much of the set theoretical work on $L(\mathbf{R})$ assuming $\mathrm{AD}^{L(\mathbf{R})}$ has been directed toward computing the ordinals $\boldsymbol{\delta}^1_n$ for $n < \omega$, which measure the definable length of $\mathbf{R}$. Precisely, $\boldsymbol{\delta}^1_n$ is the least ordinal κ so that there is no surjective $F\colon \mathbf{R} \to \kappa$ with "$F(x) \leqslant F(y)$" a $\boldsymbol{\Delta}^1_n$-definable relation of x, y. Assuming $\mathrm{AD}^{L(\mathbf{R})}$, in $L(\mathbf{R})$ the $\boldsymbol{\delta}^1_n$ are all regular cardinals, and $\boldsymbol{\delta}^1_1 = \aleph_1$, $\boldsymbol{\delta}^1_2 = \aleph_2$, $\boldsymbol{\delta}^1_3 = \aleph_{\omega+1}$, and $\boldsymbol{\delta}^1_4 = \aleph_{\omega+2}$. No upper bound for $\boldsymbol{\delta}^1_5$ is known. The problem of computing $\boldsymbol{\delta}^1_n$ in terms of the cardinals of $L(\mathbf{R})$ is interesting for a number of reasons. The definition of $\boldsymbol{\delta}^1_n$ has the same meaning in V as in $L(\mathbf{R})$, so the computation would yield an upper bound for $\boldsymbol{\delta}^1_n$ in terms of the cardinals of V. If in addition the small regular cardinals in $L(\mathbf{R})$ are regular in V, as appears possible, then one has an explicit, definable failure of the Continuum Hypothesis. Finally, the computation involves a deeper analysis of scales on projective sets (which generalize the classical sieves on co-analytic sets), and scales are of central importance in descriptive set theory.

The methods developed to attack this problem involve a complicated mix of pure set theory and definability theory. The chief tools from pure set theory are the infinite exponent partition relations $\kappa \to (\alpha)^{\beta}$ with $\beta \geqslant \omega$, the measures these generate, and ultraproducts by such measures. These tools, especially the partition relations, are Kleinberg's subject.

We now survey the book briefly.

We say $\kappa \to (\alpha)^\beta$ if given any partition of the increasing β sequences from κ into two pieces, there is an order type α subset A of κ so that all increasing β sequences from A fall in the same piece. The existence of such relations with $\beta \geqslant \omega$ follows from ZF + AD + DC; like AD itself, $\kappa \to (\alpha)^\beta$ with $\beta \geqslant \omega$ can be refuted using the axiom of choice. The chief result of Chapters 1 and 3 is that if $\kappa \to (\kappa)^{\lambda+\lambda}$, then the filter of λ-closed unbounded subsets of κ generates a normal ultrafilter μ_λ on κ; if there are fewer than κ regular $\lambda < \kappa$, then these μ_λ are the only normal ultra filters on κ. This useful result is due to Kleinberg.

Definability theory and games make their only direct appearance in Chapter 2, where ZF + AD + DC is used to show $\aleph_1 \to (\aleph_1)^{\aleph_1}$ and ${}^{\aleph_1}\aleph_1/\mu_\omega \cong \aleph_2$.

The heart of the notes is Chapters 4, 5, and 6. There Kleinberg uses the two consequences of ZF + AD + DC just mentioned to show that $\aleph_2 \to (\aleph_2)^\alpha$ for all $\alpha < \aleph_2$, while $\aleph_n$ is a singular Jonsson cardinal for $3 \leqslant n < \omega$ and $\aleph_\omega$ is Rowbottom. The proof yields a similar result with an arbitrary κ so that $\kappa \to (\kappa)^\kappa$ replacing $\aleph_1$. Kleinberg's method is an extension of that used by Martin and Paris to show $\aleph_2 \to (\aleph_2)^\alpha$ for all $\alpha < \aleph_2$. The main additional problem is to classify the finite tuples of ordinals less than $\aleph_{n+1}$ according to their representations in the ultrapower ${}^{[\aleph_1]^n}\aleph_1/\mu_\omega$. A finer (in a sense complete) classification was developed by Kunen around 1970, but his work was not published and existed only as a rumor until recovered by Solovay in [4]. From Theorem A.3 of [4] it follows directly $\aleph_n$ is Jonsson for $3 \leqslant n < \omega$, but more work seems needed to show $\aleph_\omega$ is Rowbottom. We emphasize, anyway, that Kleinberg's work was done independently of Kunen's, and published earlier.

Chapter 7 collects some results on the conflict between infinite exponent partition relations and choice. For example, Theorem 7.1 describes those relations consistent with the statement: every well ordered family of nonempty sets has a choice function. The description it gives would be complete if we knew whether $\omega \to (\omega)^\omega$ is consistent with this statement. Incidentally, there is a misprint in the statement of Theorem 7.1 which is quite confusing if (and only if) one reads the proof of the theorem with its misstatement in mind.

There are other interesting results of a combinatorial nature scattered throughout the book.

The book is written clearly; its exposition proceeds at a leisurely but not lazy pace. Proofs are often prefaced with helpful intuitive explanations. Occasional rhetorical extravagances do deserve deletion; for example, on p. 5 we are introduced to "the notion of partition relation, the single most important concept in modern large cardinal theory". (What of measures and embeddings?)

The reviewer has serious criticisms only of the book's introduction. There we meet AD as an alternative to the axiom of choice; the possibility of an inner model like $L(\mathbf{R})$ is not mentioned. AD cannot survive on this footing.

Further, Kleinberg does not mention the problem of computing δ_n^1, a problem which has motivated a great deal of the theory to which these notes contribute. As a result of these omissions, the nonexpert who does not simply love the bizarre will find little reason to read past the introduction.

How should one use the book? Curiously, although its natural context is a quite elaborate and sophisticated theory, only in Chapter 2 does the book require more than the basics of set theory. Nevertheless, the nonexpert who wants to learn something of the set theory of $L(\mathbf{R})$ assuming $\mathrm{AD}^{L(\mathbf{R})}$ would be much better advised to start elsewhere, perhaps by reading [2], [3], [4] (in that order). He will be rewarded with a broader view of this field. On the other hand, the expert in the field will find these notes a useful and well-written reference on one of its aspects.

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Several complex variables, by H. Grauert and K. Fritzsche, Graduate Texts in Math., vol. 38, Springer-Verlag, New York, Heidelberg, Berlin, 1976, viii + 206 pp.,$18.80.

There is no shortage of introductory texts and treatises in several complex variables. Since the subject borrows methods from diverse areas of mathematics such as algebraic geometry, functional analysis, p.d.e., differential geometry and topology, and thus partially overlaps with these areas, it is not surprising that these introductory books cover a wide variety of material from various perspectives. [A listing of some of the recent introductory texts (in English) in several complex variables is given in the references below.] Grauert and Fritzsche's *Several complex variables* is a more or less orthodox introduction to the classical themes of several complex variables arising out of the Oka-Cartan theory. The classical global theory of functions of several complex variables is based on the existence, noted by Hartogs, of domains D in $\mathbf{C}^n$ ($n > 1$) such that every holomorphic function on D can be extended to a larger domain. If on the contrary D is the natural domain of definition of a holomorphic function, then D is called a *domain of holomorphy*. The subject received its major impetus from attempts to describe domains of holomorphy. The Cartan-Thullen Theorem characterizes domains of holomorphy as domains D that are *holomorphically convex*; that is, the hull of any compact subset with respect to the algebra of holomorphic functions on D is compact.