to the proof. The reviewer finds that kind of organization unattractive–not to say Teutonic. A lemma at the appropriate place would be more to the point.

It is natural to compare the Brown-Pearcy text with that of Reed and Simon, since both pairs of authors regard functional analysis as a way-station on the road to operator theory. There is, fortunately, little similarity in the approach of the two texts. Reed and Simon treat the subject like an unpleasant chore, and their text is definitely not suitable for a general functional analysis course. One of the really unpleasant features of the Reed-Simon book is that many of the proofs which are not completely straightforward are simply omitted. By contrast, Brown and Pearcy treat both the subject matter and the reader with appropriate respect.

Professors Brown and Pearcy write with a very nice pedagogical style. Without being prolix, they do enough chatting so a student can tell what's going on. They give an idea of how important a result is, for example, and why the other half of an "if and only if" theorem is somewhere else in the book. They do not assume that a student has all possible details at his fingertips (e.g. absolute convergence implies convergence in a Banach space–where this is used a reference is given to the problem where the proof is outlined). The authors talk about the equivalence class versus function dilemma in L_p enough so the student will know that subsequent lack of precision is both deliberate and appropriate.

The reviewer does have some nit-picking complaints. The text has an extensive list of cross references from problem to theorem to example, which is good. However, it badly needs a list of special symbols. The terminology "unital algebra" (that's an algebra with unit) is surely a barbarism of the worst sort. The indexing is quaint. The Springer format is unlovely.

Aside from a few cavils like those above, the reviewer feels that this book is really first rate, and will serve well both as a text for a standard graduate course and as a reference work.

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Boolean-valued models and independence proofs in set theory, by J. L. Bell, Clarendon Press, Oxford, 1977, xviii + 124 pp., $14.50.

This lucid and elegant introduction to Boolean-valued models is a book which could have been written ten years ago, and in fact was supposed to have been–by different authors! As Dana Scott says in his introduction to the book, it "essentially supplants" a projected paper by Scott and Robert Solovay which may be one of the most-referenced unwritten papers in recent mathematical history. The contents of that nonexistent paper (and this book) are as familiar to the set theorist today as is sheaf theory to the algebraic geometer, but clear and detailed expositions for the beginner are still relatively scarce, so this book is welcome.

For those not familiar with the recent history of set theory a few words of background may be in order. In the years B.C. (Before Cohen) there were very few standard models of set theory (i.e., models in which the $\in$-symbol is interpreted by the standard membership relation) for the set theorist to study: the principal one–besides the universe of all sets–was Gödel's class of constructible sets which he used to prove the relative consistency of the Axiom of Choice (AC) and the Continuum Hypothesis (CH) with ZF, the basic Zermelo-Fraenkel axioms of set theory. It was well known that the general method of "inner models" which Gödel used–i.e., cutting down the universe to a definable class such as the constructible sets–could not be used to prove the relative consistency of the negation of AC or CH. It was natural to expect that nonstandard models would be needed for the proof of these results. But Paul J. Cohen found a way to expand a countable standard model (countability was essential as he observed) in a controlled and economical fashion into a standard model in which CH or AC is false. Scott says, "I knew almost all the set-theoreticians of the day, and I think I can say that no one could have guessed that the proof would have gone in just this way". (All of the following quotes are from Scott's introduction.) Cohen's method was so flexible that it was able to produce an abundance of models of set theory. "We had no idea before Cohen (and those who so quickly jumped into the field after him) how much independence there could be. Thus we can make (if anyone would want to) $2^{\aleph_0} = \aleph_{17}$ and $2^{\aleph_1} = \aleph_{2001}$ in some model or other, and even with these silly choices $2^{\aleph_2}$ is not at all determined (except that it has to be greater or equal to $\aleph_{2001}$ and has to avoid certain singular cardinals)". Moreover Cohen's method of forcing has by now been used to prove independence results not just in cardinal arithmetic, but in areas of analysis, algebra, and topology seemingly far removed from the foundations of mathematics.

"The idea of using Boolean-valued models to describe forcing was discovered by Solovay in 1965. He was using Borel sets of positive measure as forcing conditions; the complications of seeing just what was true in his model led him ... to summarize various calculations by saying that the combination of conditions forcing a statement added up to the 'value' of that statement ... Vopenka independently had much the same idea". Scott and Solovay realized that "by *starting* with Boolean-valued sets from the beginning, many of the more tedious details of Cohen's original construction of the model were avoided". In the Boolean-valued approach one starts with any model M of set theory (countable or uncountable) and any complete Boolean algebra B and constructs a model $M^{(B)}$ in which any statement φ about the "sets" in $M^{(B)}$–such as "$a \in b$", or "$a = b$", or CH–is assigned a "truth value" $[\![\varphi]\!]$, which is an element of B. It may then be proved that every axiom of ZF receives truth value 1, and that, for an appropriate choice of B, $[\![\mathrm{CH}]\!] = 0$; since the rules of inference preserve truth value 1, it follows that CH is not derivable from ZF. From this point of view to say that an element p of the Boolean algebra B "forces" a statement φ about sets means that $p \leqslant [\![\varphi]\!]$. Moreover if M is countable one can use the Rasiowa-Sikorski theorem to obtain an ultrafilter U in B such that "factoring out" by U yields a $\{0, 1\}$-valued (i.e., ordinary) model $M[U]$ such that a statement φ is true in $M[U]$ if and only if there is a $p \in U$ which forces φ. Thus, "as was demonstrated by the paper of Shoenfield [3] ... there is very little to choose between the methods: forcing and Boolean-valued models both come to the same thing." However, some perspective is gained by viewing the method from these two slightly different points of view; thus for example Shoenfield's construction of the model is illuminated when the Boolean-valued point of view underlying it is explicitly understood; and on the other hand the choice of the particular set of forcing conditions, or Boolean algebra, to use in constructing a model where, for example, CH fails is better motivated in Shoenfield than in Bell.

The book under review is one of a new series called the Oxford Logic Guides, whose stated purpose is "to encourage interdisciplinary study of the areas of mathematics, philosophy, history and philosophy of science and linguistics, in which formal or mathematical logic can play a role. It will provide materials for the study of logic at an intermediate level, in an attempt to bridge the gap between elementary and advanced books . . . " Bell's book admirably fulfills these purposes; it is easily accessible to anyone with a basic knowledge of axiomatic set theory including constructible sets, and is suitable either for self-study or a beginning course in set theory. (It includes a number of exercises.) I used it recently in a small seminar with graduate students who gave talks based on the book; the students found the book well-motivated and easy to understand. (By way of comparison, they found Jech [2] too sketchy.) The book seems to be remarkably free of errors. (In Theorem 2.7 it is sufficient to assume that $|B| \leqslant 2^{\aleph_0}$, which simplifies the proof of 2.8.) The attributions of theorems are often not to the original author but to the author of an expository source (such as Scott's widely circulated notes); both the preface and the introduction warn about this, but I found that this is still misleading to the student.

Besides the original results of Cohen, the book contains Solovay's proof, using forcing, of the Gaifman-Hales theorem that there are Boolean algebras of arbitrarily large cardinality which are the completion of a countable subset. But aside from the inclusion of some recent results of Bell on generic ultrafilters (which only serves to complicate the presentation of Chapter 4) this slim volume could indeed have been written in 1967: there is no mention of Martin's Axiom, Suslin's problem, Kurepa's hypothesis, or Lebesgue measurability; Easton's theorem is mentioned but not proved. The mathematician who reads this book or Shoenfield's article to acquaint himself with the basic techniques of forcing and who wishes to pursue his study further would do well to turn next to the excellent expository article by Burgess [1] or the lecture notes by Jech [2].

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