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Theory of modules, by Alexandru Solian, John Wiley & Sons Limited, London, New York, Sydney, Toronto, 1977, x + 420 pp., $26.50.

Somewhere between saying too little and saying too much lies good exposition. Most of the pitfalls are located to one side or the other of that rather narrow ridge where the essential ideas are provided without a deluge of trivialities. Being one who tends to fall off the ridge at regular intervals, it interests me to speculate on the reasons behind difficult lecturing or writing styles. One reason, of course, is inexperience, and I believe that criticism of exposition is an important part of graduate education. In seminar presentations, I feel that students are too often let off the hook because what they are doing is mathematically correct, even though what they are saying may be devastating for the understanding of the other participants. However teaching someone to teach is difficult, and perhaps dangerous too, if one is not absolutely sure of the difference between what enlightens and what confuses. Let us consider some of the other possible reasons behind incomprehensibility.

In my early years I was aware that I invariably understood some people and rarely understood others, without attributing this to any particular qualities of those involved. It was only later that I realized that those whom I could follow tended to be secure individuals, with enough self-confidence to tell me something I already knew, or remind me of something I knew a week ago. We are probably all a little sensitive to the reply, "But that's trivial," especially when it concerns something which we have found anything but trivial, and perhaps those who are least affected by the reply are by and large those who refrain from using it. When someone begins an explanation by assuming that his audience is plunged into the matter as deeply as he is, I usually feel that he is protecting himself from something. But of course insecurity is not always the reason for a bulldozer style. Sometimes it is a simple matter of insensitivity, an inability to realize that others are not

following, or are not interested. And sensitizing someone to this kind of thing is probably even more hopeless than teaching him how to teach. Let's consider the opposite phenomenon, which is really the one that concerns us here, namely the tendency to say too much.

Again it is probably a universal temptation to take a body of material which has not come easily to us, and, like good neophytes, set about an exposition which will contain every excruciating thought we ever had on the subject. In the early sixties, for example, when we were being told that progress was directly proportional to the number of mathematicians in the world, there was a movement to accomplish the increase in population by writing freshman textbooks which spared neither an epsilon nor a delta. The result was that not only did freshmen not learn epsilons and deltas, but many of them didn't even find out what a derivative was. (I recall the definition in one book I was using as

$$f'(a) = \lim_{0} \frac{f \circ (I + a) - f \circ a}{I}$$

where I is the identity function, and a is not really a, but rather the constant a-valued function.) The effect of such excessive concern for wear and tear on the mind of the reader is soporific. Rather than being grateful for our attempt to do his thinking for him, he is more likely to start counting the pages before the end. One is usually content that two plus two is four, unless a reference is provided, in which case one is sometimes tempted to peek just in case he may be overlooking something, only to find that some lemma of an earlier chapter states that two plus two is in fact four, usually with a reference to some other lemma. This concern for detail arises undoubtedly from a fear of not having properly mastered the material already covered. It nips seminars in the bud, and elevates the routine to a position of central importance. One becomes satiated on appetizers and never gets to the main course.

That this tendency prevails in the present volume is already indicated on the title page, where one wonders just how much the subtitle, "An Introduction to the Theory of Module Categories" adds to the main title "Theory of Modules." More evidence is occasioned in the numerous footnotes. Examples: "Here we replaced, by an abuse of notation, $\lim E_\lambda$ by $\lim M_\lambda$, taking into account that $E_\lambda = M_\lambda$, $\lambda \in I$" (page 222). "We denoted by $\{\lambda\}$ the subset of I containing only the element λ" (page 248). In a footnote on page 154 it is explained that by the direct sum of E with E, we really mean the direct sum of E_1 with E_2, where $E_1 = E$ and $E_2 = E$. Another one on page 349 informs us that the term *essential extension* applied to a monomorphism $E \xrightarrow{u} F$ is a discrepancy of terminology, since we should really say "F is an extension of coker u by E." On page 108 it is shown in considerable detail how to apply a homomorphism to an infinite sum of finite support.

To illustrate the mileage which can be gotten from a simple fact if one leaves no stone unturned, it takes six pages to show that Hom commutes with limits, including a proposition which takes two thirds of a page to state. To be fair, it is done with limits simultaneously in both variables, but with the

amount of categorical exposition that has already taken place (this is page 259), one tends to wonder just what is going on. Likewise, it takes three quarters of a page to state that the Yoneda imbedding $C \mapsto \mathrm{Hom}(C,)$ is full and faithful, and another page to prove it, even though it is a corollary of the natural isomorphism

$$\mathrm{Nat}(\mathrm{Hom}(C,), F) \simeq FC$$

which has been established earlier in the chapter.

The topics covered tend to be standard preliminary notions of module theory. There is also a good deal of category theory, and indeed one of the objectives is to explain the notions of limit, adjoint functor, and abelian category by first doing them for modules, and then indicating how they generalize. The most notable omission is the tensor product, which, it is stated in the preface, may be the subject of a future volume. However one wonders if a 400 page introduction to module theory can really be considered as such, without a single mention of what is perhaps the most important notion in the subject, with the possible exception of Hom. The omission seems all the more significant in a book which purports to teach adjoint functors.

I shall also air a prejudice with regard to the use of abelian categories. In the early days, one was naturally intrigued by the possibility of doing things by arrows alone, and one was willing to go through considerable acrobatics to prove what, in the module case, was child's play. However, if abelian categories stuck, it was not so much because of the applications to non-module categories (sheaf cohomology notwithstanding), but rather because the proofs became so simple that, with duality at hand, it became cleaner and shorter to write them down in the more general case. If I still introduce abelian categories in a course in homological algebra, it is not with the idea of making converts to category theory, but rather with the intention of speeding things up. In this book, however, they slow things down. What may seem at the outset as good pedagogy, namely, introducing a notion in the concrete case and then indicating how it can be abstracted, is just another obstacle in the way of getting to anything really interesting. One should decide in advance what level of maturity one is aiming at, and either work abelian categories to death, or else chuck them altogether.

The last chapter contains an account of the full imbedding theory for small abelian categories. The Gabriel-Popesco theorem is also stated here, but without proof, which is unfortunate, since a short proof by Takeuchi (J. Algebra **18** (1971), 112–113) makes it quite easy. In fact, the full imbedding theorem is the only theorem of some depth proved in the book. It is understandable that a work which has tried to give a parallel treatment of modules and abelian categories should want to culminate with a theorem which says that, in a certain sense, from now on you only have to worry about modules. I am flattered by such a role for the theorem, which has been described in the preface as one of real mathematical beauty. However in all objectivity, I must admit that the tensor product strikes me as more important to the understanding of basic module theory than the full imbedding theorem.

The above may read as a little harsher than I had intended. I have no desire to demolish a work which was obviously done with great care and over a long

period of time. I found no errors, either mathematical or typographical, in what was admittedly a rather sporadic reading. In spite of all the detail, the format of the printed page does not tend to oppress, and the formulas and diagrams are well displayed. I should also add that I don't get universal approval when carrying on in the above strain. In my department, for example, I try to push Lang's book "Algebra" as a first year graduate algebra text. It seems to me that it proceeds at an ideal pace, and covers a great deal of material, focusing attention as it does on key ideas, and leaving to the reader what the reader should have left to him. However the book is hardly ever used here, since I am told by my colleagues that the students prefer a book in which everything is done for them. The result is that we invariably use a book in which less material is covered in more space, where subscripts flourish on superscripts, and where little attempt is made to distinguish the important from the routine. Thus I can be reasonably sure that there are those who would prefer the present book to anything I might recommend for such a course. Nevertheless, I maintain that when an author is tempted to include a minor verification, he should ask himself for whom he is doing it. If he is simply satisfying himself that the point is trivial, then he should omit it. But if he thinks he is doing some potential reader a service, he might better consider advising that reader that he is out of his depth. Traffic should not be slowed for the pedestrian walking down the white line. It should rather be suggested that he would be happier on the sidewalk.

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Categories of algebraic systems, by Mario Petrich, Lecture Notes in Mathematics, vol. 553, Springer-Verlag, Berlin, Heidelberg, New York, 1976, viii + 217 pp., $10.20.

Mal'cev varieties, by Jonathan D. H. Smith, Lecture Notes in Mathematics, vol. 554, Springer-Verlag, Berlin, Heidelberg, New York, 1976, viii + 156 pp., $7.40.

> . . . ask not what your country can do for you;
> ask what you can do for your country.
>
> J. F. Kennedy

1. I was introduced to the concept of a category around 1960 by A. G. Kurosh. He pointed out that the origin of a segment of category theory and a part of lattice theory was in the observation that many results on direct decomposition of groups (e.g., the Kurosh-Ore Theorem and the Ore Theorem) depend very little on the structure of the group. The proofs can be stated very simply in terms of homomorphisms of groups and their properties (that is, in a categorical language) or in terms of the set theoretic inclusion among the normal subgroups of a group (that is, in lattice theoretic language).

Category theory was started by S. Eilenberg and S. Mac Lane [2], [3] as a