form and later by Schmidt (1907) in connection with reflection and transmission of light through inhomogeneous media. Ambarzumian (1943) articulated the approach more clearly and exploited it further, but it was Chandrasekhar who, in his well-known book *Radiative transfer* (Dover, New York, 1960, first published in 1950) stated in full generality the idea described above and called it "principles of invariance". Chandrasekhar proceeded in his book with the exploitation of the principles of invariance extracting a phenomenal amount of information and solving several problems in transport theory that were considered impossible to solve up to the late 1940's. In an interesting article (J. Mathematical Phys. **41** (1962), 1–41), R. M. Redheffer gives a lucid survey of some of the early developments, as well as his own contributions, up to 1962. The authors of the present book have participated vigorously in the exploitation of the imbedding idea in a variety of contexts and have written several books and papers on it and its applications in addition to the present one.

The first four chapters explain in a very simple context the basic idea of imbedding and the mechanics of converting boundary value problems to initial value problems. The remaining eight chapters deal with more specific applications such as random walks, wave propagation, calculation of eigenvalues, WKB and integral equations. Many of the devices and techniques presented in this book may be known to a lot of mathematicians, perhaps by different names (or no names at all). The purpose of the book is, however, to reach a wide audience and this will no doubt be accomplished because the writing is lucid and with a lot of redundancy to make reading easy.

Nevertheless it seems that too much space is occupied with generalities while not enough substantial examples are treated. This should be contrasted with Chandrasekhar's book where the general ideas take up a few pages and nearly half the book is devoted to detailed analysis and computations.

Mathematicians may find interesting, however, the several possible applications of the imbedding idea and, one never knows, it may be useful in their own research also.

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Cohomology theories for compact abelian groups, by K. H. Hofmann and P. S. Mostert, Springer-Verlag, New York, 1973, 236 pp., $18.50.

Every compact abelian group is a projective limit of compact abelian Lie groups, each of which is the product of a torus and a finite group. That is, one obtains the most general compact abelian group by starting with the circle group and the finite cyclic groups, then taking products and limits. Thus, in studying invariants for compact abelian groups one should often be able to compute an invariant in general once one knows it for these simple building blocks and understands how it is treated by the product and limit operations.

This principle is one of the main themes of the book. It first arises in the computation of the Čech cohomology ring $H(G, R)$ of a compact abelian group—a project carried out in an earlier paper of Hofmann and reproduced in this book. However, this application is relatively easy and occupies only a small portion of the book. The major portion is devoted to the study of another functor $h(\cdot, R)$ on compact groups which is susceptible to the same methods. We attempt to describe this second "cohomology theory" below.

Čech cohomology is a functor on topological spaces. Applied to compact groups it sees only the topology and ignores the algebra. There is also a cohomology theory for groups which is purely algebraic and ignores the topology when applied to compact groups. Briefly, this is constructed as follows: Given a group G, one constructs the integral group ring $Z[G]$. One then considers Z to be a $Z[G]$ module via the standard augmentation and finds a free $Z[G]$ module resolution of Z. The algebraic cohomology of G with coefficients in a $Z[G]$ module R is then obtained by applying $\operatorname{Hom}(\cdot, R)$ to this resolution and passing to the cohomology of the resulting cochain complex.

The cohomology theory for compact abelian groups studied by Hofmann and Mostert agrees with the above algebraic theory on finite groups. However, for general compact abelian groups G it takes into account both the algebra and the topology of G. A discussion of this theory requires a brief digression into the theory of principal bundles.

Let G be a topological group and E a space on which G acts continuously (on the right). We assume that for each $x \in E$ the map $g \to x \cdot g$ is a homeomorphism of G onto the orbit containing x. Let B be the decomposition space obtained by identifying orbits of the action to points and let $\pi: E \to B$ be the projection. We assume that π has local cross sections; i.e., for each $y \in B$ there is a neighbourhood U of y and a map $f: U \to E$ such that $\pi \circ f = \mathrm{id}$. Then (E, B, π) is called a principal G bundle over B. Two such bundles (E_1, B, π_1) and (E_2, B, π_2) (with the same base B) are called isomorphic if there is a homeomorphism $E_1 \to E_2$ which commutes with the action of G.

If G is a compact topological group, there is an essentially unique principal G bundle $E(G) \to B(G)$ for which $E(G)$ is contractible. This is the universal bundle for G. The space $B(G)$ is called a classifying space for G. There are standard constructions of classifying spaces which yield $G \mapsto B(G)$ as a functor from compact groups to topological spaces (due to Milnor, Dold-Lashof, and Milgram for example). The functor studied by Hofmann and Mostert is defined by $h(G, R) = H(B(G), R)$,—that is, $h(G, R)$ is the Čech cohomology with coefficients in R of a classifying space for G.

Classifying spaces arise in a number of contexts. They are used to classify principal bundles over a Lie group G. That is, there is a bijective correspondence between isomorphism classes of principal G bundles over a space X and homotopy classes of maps from X to $B(G)$. In particular, when G is the $n \times n$ unitary group $U(n)$, $B(G)$ classifies the n-dimensional complex vector bundles over X. When G is a discrete abelian group, the set of homotopy classes of maps from X to $B(G)$ is in bijective correspondence with the first Čech cohomology group of X with coefficients in G, that is, $B(G)$ is an Eilenberg-Mac Lane space of type $K(G, 1)$ in this case. Also, for finite abelian groups, the Čech cohomology $h(G, R)$ of $B(G)$ agrees with the algebraic cohomology of G with coefficients in R, where R is considered a trivial G module.

Because of the connection with bundle theory, Eilenberg-Mac Lane spaces, and cohomology of finite groups, it is reasonably clear that an understanding of classifying spaces and their Čech cohomology groups is important—at least for Lie groups and, in particular, for finite groups. Whether such an understanding has great significance for general compact abelian groups is not made clear by this book. In any case, the main objective of the book is to compute, as explicitly as possible, the cohomology ring $h(G, R)=H(B(G), R)$ for arbitrary compact abelian groups G and coefficient rings R. For connected G this is a relatively simple matter. Here the answer is that $h(G, R)\simeq R\otimes P\hat{G}$ where $P\hat{G}$ is the integral polynomial ring on the dual group $\hat{G}$ of G and elements of $\hat{G}$ are given degree 2 (so that $P\hat{G}$ is a graded ring where all nontrivial homogeneous elements have even degree). That this is the correct answer if G is a torus is well known and easy to prove. Since compact connected abelian groups are projective limits of tori, the general result follows from a functorial limit argument. Thus, the difficulties in computing $h(G, R)$ in general are due to the disconnected case. In fact, the crux of the matter is the finite case. Although finite abelian groups have a very simple and well-understood structure, their cohomology rings apparently have a surprising resistance to explicit computation.

Since, for finite abelian G, $B(G)$ is an Eilenberg-Mac Lane space $K(G, 1)$, the problem of computing the group $h(G, R)$ is a special case of the problem of computing the Čech cohomology of the general Eilenberg-Mac Lane space $K(G, n)$. The latter problem is studied in the 1954/55 Seminar Cartan. However, the computations for $K(G, 1)$ in Hofmann and Mostert are more detailed and explicit. This computation of the cohomology of finite abelian groups is perhaps the most interesting part of the book, so we describe it in some detail below.

Let G be a finite abelian group and $Z[G]$ the integral group ring of G. The cohomology of G is computed by using a particular free $Z[G]$ module resolution of Z, which is constructed as follows: let $0\rightarrow F\xrightarrow{f}F\xrightarrow{\pi}G$ be a free resolution of G, where F has generators $x_1, \cdots, x_n$ and $f(x_i)=z_ix_i$. Let X be the graded algebra $Z[G]\otimes PF\otimes\Lambda F$, where PF (resp. ΛF) is the polynomial (resp. exterior) algebra generated by F with elements of F given degree 2 (resp. degree 1). There is an obvious augmentation $X\rightarrow Z$ and a differential $d+\partial$, where d and ∂ are $Z[G]$ module derivations characterized by

$$d(1\otimes x_i\otimes 1)=\bar{z}_i\otimes 1\otimes x_i \qquad (\bar{z}_i=\pi(x_i)+\pi(x_i)^2+\cdots+\pi(x_i)^{z_i}),$$

$$d(1\otimes 1\otimes x_i)=0, \qquad \partial(1\otimes x_i\otimes 1)=0,$$

and

$$\partial(1\otimes 1\otimes x_i)=(\pi(x_i)-1)\otimes 1\otimes 1.$$

It turns out that $X\to Z$, with this differential, is a resolution of Z by free $Z[G]$ modules. Thus, $h(G, R)$ is the cohomology of the cochain complex $\mathrm{Hom}(X, R)$. The separate gradations in PF and ΛF give this complex the structure of a bigraded module with a differential of bidegree $(2, -1)$. Hence, it can be regarded as the E_2 term of a spectral sequence whose E_3 term yields the desired cohomology. It is this point of view which the authors exploit to obtain their results.

Using the edge terms of the above spectral sequence, one obtains a morphism

$$P_R\,\mathrm{Ext}(G, R)\otimes_R \mathrm{Hom}(\Lambda G, R)\to h(G, R)$$

in the case where G acts trivially on R. This is an isomorphism in certain important cases (e.g. when R is a field). For integral coefficients the situation is apparently not completely understood. The best result obtained here says that $h(G, Z)$ is a torsion free $P\hat{G}$ module with a minimal generating graded subgroup $M(G)$ which is naturally isomorphic to $\Lambda\hat{G}$ with a degree shift.

The above is only a brief sampling of the material on cohomology of finite abelian groups. At least half of the book is devoted to the algebraic machinery that goes into these results.

Having disposed of finite groups (to the extent possible) the authors go on to a description of classifying spaces and the definition of $h(G, R)$ for general compact groups G. Included are a discussion of the Milnor construction of classifying spaces and a proof that for finite abelian groups the algebraic cohomology agrees with the Čech cohomology of a classifying space.

With the information obtained about $h(G, R)$ (and that trivially available about the Čech cohomology $H(G, R)$) for finite groups and for tori, one obtains analogous information, in general, by using the fact that every compact abelian group is a projective limit of products of finite groups with tori. However, one must exercise a certain amount of care regarding functorial considerations. This aspect of the problem is placed in the context of a general discussion of Kan extension of functors which may be of independent interest to algebraists and category theorists.

As a sampling of results obtained by these methods one has that for a general compact abelian group G with identity component G_0:

$$H(G, R)\simeq R\otimes C(G, Z)\otimes\Lambda\hat{G}_0$$

where $C(G, Z)$ is the space of continuous Z valued functions on G, and in the case of field coefficients

$$h(G, R)\simeq R\otimes PG\otimes\Lambda\,\mathrm{Tor}(\hat{G}, K)$$

where K is the prime field of R.

It should be obvious by now that Hofmann and Mostert's book is a monograph with a rather narrow objective and it should attract a correspondingly narrow audience. The audience may be narrowed further by the fact that the book is written in a difficult and demanding style. The authors apparently assume that the reader has a rather sophisticated background in category theory, homological algebra, algebraic topology, and the structure theory of compact abelian groups. Difficult and sometimes obscure pieces of background information are often used implicitly without reference and without a decent warning to the reader. Statements of theorems often take up whole pages while the proofs are tossed off with a few casual remarks. If the reader attempts to read the book in the order it is presented, he is forced to hack through great jungles of unmotivated technical material with little idea of its objective (this is particularly true in the first half of the book). The authors have a tendency to develop technical background material in the greatest possible categorical generality. In some cases this may have definite value for certain specialists, but we suspect most readers will find that it diverts in an annoying way from the main objective of the book and adds greatly to the time and effort needed to understand it.

Clearly this book is for the highly motivated reader who is well trained in homological algebra and has a definite need for explicit computations of $h(G, R)$.

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