technique was extended by Cesari in 1966, who found existence theorems for optimal controls which are more widely applicable.

The first of two chapters on existence theorems is devoted to the Filippov-Cesari technique, and to further results of Berkovitz. These results all depend on certain convexity assumptions. When these assumptions do not hold, there may be no optimal control in the usual sense of the term. The second chapter on existence is devoted to existence of generalized optimal controls without convexity assumptions. The basic idea of generalized solutions to problems in calculus of variations first appeared in L. C. Young's work during the 1930's, under the name "generalized curves". It reappeared in control theory under the names "sliding regimes" or "relaxed controls", in work of Gamkrelidze, Warga, McShane, and others.

In summary, this book gives a well-presented treatment of a well-selected package of topics. It is a valuable addition to the control theory literature.

W. H. FLEMING

Einige Klassen singularen Gleichungen, by S. Prössdorf, Akademie-Verlag, Berlin; Mathematische Reihe, Band 46, Birkhäuser, Verlag, Basel and Stuttgart, 1974, 353 pp.

An important early chapter in operator theory was concerned with the study of various classes of integral equations with singular kernel. Chief among these were F. Noether's investigation of integral equations with Cauchy kernel and the study by N. Wiener and E. Hopf of the integral equation which now bears their name. The results of these studies were not only of considerable practical importance but were instrumental in the development of the abstract notions of Fredholm operator (called Noether operator in the book under review) and index. The study of various spaces of analytic functions was stimulated, as well as the problem of factoring functions on the real line into factors analytic on the upper and lower half-planes. In addition, the important notion of the symbol of an operator first occurred in this context.

In the book under review the author presents the theory of various classes (including systems) of one-dimensional singular integral operators including those with Cauchy or Hilbert kernels as well as the study of Wiener-Hopf and Toeplitz operators. In the definition of these operators, a function called the symbol can be identified which determines the behavior of the operator up to "smooth" operators. In particular, the operator with symbol equal to the inverse of the symbol of the given operator defines an inverse for the operator modulo the compact operators; thus the problem of deciding when an operator is Fredholm is reduced to inverting the symbol. The techniques are drawn largely from functional analysis and the methods show the strong contribution that Soviet mathematicians have made to this subject, especially I. C. Gohberg and his collaborators.

In addition to the more common results on operators of normal type which arise when the symbol is pointwise invertible, the author presents an intensive study of what happens when the symbol is permitted to have certain kinds of zeros. Briefly, zeros are allowed which arise when the function can be factored into the product of some canonical function (usually a polynomial) with a nonvanishing continuous function. The operator defined by the canonical function is studied in considerable detail usually by putting an appropriate norm on the range relative to which the operator is well behaved. The study of the general case is then reduced to this.

A number of books have appeared recently on related topics. Perhaps the closest is the book of I. C. Gohberg and I. A. Fel'dman, *Convolution equations and the projection method for their solution*, "Nauka", Moscow, 1971; Amer. Math. Soc., Providence, R.I., 1974. Much of the material on "abstract" singular integral operators mentioned above also appears in this book including a treatment with considerable detail of the solution of such equations by the projection method. In the book of Prössdorf, the latter topic is discussed in a short appendix. Another related book is by I. C. Gohberg and N. J. Krupnick, *Introduction to the theory of one-dimensional singular integral operators*, "Shtintsa" Kishiev, 1973, which takes up the study of singular integral operators on contours in considerable detail with coefficients from a variety of spaces. Again there is considerable overlap, but operators with discontinuous coefficients are treated by Prössdorf only in an appendix. Lastly, there are two books by the reviewer, *Banach algebra techniques in operator theory*, Academic Press, New York, 1972, and *Banach algebra techniques in the theory of Toeplitz operators*, (CBMS Regional Conference Series, No. 15) Amer. Math. Soc., Providence, 1973, which confine attention to the study of Toeplitz operators on Hilbert space with all kinds of coefficients.

The book under review presents a modern unified treatment of these classes of operators. It is readable and complete on the topics it emphasizes. Its chief virtues are the material on operators not of normal type, a treatment of Fredholm theory in Fréchet spaces, and a lengthy bibliography of the area, especially of the Soviet literature which is sometimes difficult to obtain.

R. G. DOUGLAS

Lectures in functional analysis and operator theory, by Sterling K. Berberian, Springer-Verlag, New York, 1974, ix+345 pp., $14.80

Functional analysis, a short course, by Edward W. Packel, Intext Educational Publishers, New York, 1974, xvii+172 pp., $10.00

Functional analysis, by Walter Rudin, McGraw-Hill, New York, 1973, xiii+397 pp.

I liked these three books and enjoyed reviewing them. My viewpoint when reading these books was most definitely not that of a student. I did not read all (or even most) proofs in detail, nor did I carefully check formula references