BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 77, Number 2, March 1971

BOOK REVIEWS

Maximum principles in differential equations by M. H. Protter and H. F. Weinberger. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1967. x+261 pp. $8.00.

This book is devoted to the study of maximum principles in partial differential equations. It contains a wealth of material much of which is presented for the first time in a book form. An attractive feature of the book is that it is completely elementary and thus accessible to a wide audience of readers.

The book has four chapters. Chapter I deals with the one dimensional maximum principle. The discussion of this very simple model of a maximum principle forms a good introduction to the general theory. Various applications of the principle are given to show that it is a very useful tool even in the study of ordinary differential equations. As an example, it is shown that many oscillation and comparison results in the Sturm-Liouville theory could be deduced most easily by a maximum principle argument.

The proper discussion of maximum principles in partial differential equations begins in Chapter II. This chapter, which is the backbone of the book, is devoted to elliptic equations. The material covered in this chapter includes the E. Hopf maximum principle and its generalizations; the Phragmèn-Lindelöf principle for solutions of elliptic equations; Serrin's version of the Harnack inequality for solutions of general elliptic equations in two variables (this is probably the most difficult result discussed in the book); various versions of the Hadamard three circles theorems for solutions of elliptic equations; applications of the maximum principle to nonlinear equations and to problems of fluid flow.

Chapter III is devoted to parabolic equations. The plan of this chapter parallels that of Chapter II. The topics discussed include the L. Nirenberg strong maximum principle; a three curves theorem with an interesting application to the Tikhonov uniqueness theorem; a Phragmèn-Lindelöf principle for parabolic equations with applications to uniqueness results; nonlinear operators; a maximum principle for certain parabolic systems.

The fourth and the last chapter is devoted to hyperbolic equations. The results in this chapter are somewhat special since a maximum principle in the proper sense does not hold for solutions of hyperbolic equations. Nevertheless, solutions of certain hyperbolic equations which satisfy suitable initial or boundary conditions are known to exhibit a maximum principle property. A detailed study of this phenomenon (mostly in the two variables case) is the subject matter of the last chapter.

In conclusion, the book gives a very readable account of the role of maximum principles in differential equations. It should be read by anyone interested in this elementary yet basic and fascinating subject. The book which contains many examples and exercises is also very suitable as a text book.

SHMUEL AGMON

An introduction to number theory by Harold Stark, Chicago, Markham Publishing Co., 1970.

This book, according to the preface, is intended for future high school and junior college mathematics teachers, rather than for budding research mathematicians. As a result, the book has a somewhat different tone from many other texts on elementary number theory. For one thing, it is a lot more fun to read.

Much of the material in the book is fairly standard; besides an introductory chapter, there are chapters on the Euclidean algorithm and its consequences, on congruences (through primitive roots), and on some simple Diophantine equations. A chapter on rational and irrational numbers concludes with Liouville's theorem (plus a list of later results). The last two chapters, on continued fractions and quadratic fields, are somewhat harder. The continued fraction chapter starts off easily enough, but concludes with material on periodic continued fractions and Pell's equation; the material on quadratic fields amounts to a brief introduction to the phenomena present in algebraic number fields. There is one other chapter, on magic squares. Stark gives general procedure (the uniform step method) for putting numbers in squares, and then gets conditions under which the resulting square is magic. I was somewhat bothered by the definition of "magic"; it seems to me that any magic square worthy of the name should have the main diagonals add up to the magic sum, but Stark imposes conditions only on the rows and columns. His method makes the analysis easier, though. Stark does discuss diabolic (or pandiagonal) squares, in which all diagonals (main and broken) add up to the magic sum. There are a few other eccentricities in his treatment; for instance, he does not require that the numbers in a magic square lie in separate squares.

One topic which is conspicuously absent is quadratic reciprocity. Stark states in his preface that he feels it is a topic better left out;