

RIGIDITY OF HYPERSURFACES OF CONSTANT SCALAR CURVATURE

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Let M_n and \tilde{M}_{n+1} be Riemannian manifolds of dimension n and $n+1$ respectively.

Assume M_n isometrically immersed in \tilde{M}_{n+1} .

If each point p of M_n is contained in an open neighborhood $U \subset M_n$ (which may depend on p) such that no open submanifold of U is rigid in \tilde{M}_{n+1} , then M_n is called *locally deformable* in \tilde{M}_{n+1} .

This concept allowed us to show that the result of Nagano-Takahashi [3] holds without any restriction, i.e. that any homogeneous hypersurface of the Euclidean space E_{n+1} is isometric to the Riemannian product of a p -dimensional sphere and an $n-p$ dimensional Euclidean space.

This result is a consequence of the following

THEOREM 1. *Let M_n be a complete Riemannian manifold of dimension $n \geq 3$, with constant scalar curvature $K \neq 0$.*

If M_n is locally deformable in the Euclidean space E_{n+1} , then it is isometric to the Riemannian product of a 2-sphere of radius $1/K$ and an $n-2$ dimensional Euclidean space.

The next result gives rigidity of certain hypersurfaces of non-Euclidean space forms.

THEOREM 2. *Let M_n be an n -dimensional Riemannian manifold $n \geq 4$, with constant scalar curvature K . Assume M_n isometrically immersed in the space form $\tilde{M}_{n+1}(c)$, with $c \neq 0$ and $K \neq n(n-1)c$. Then M_n is rigid in $\tilde{M}_{n+1}(c)$.*

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