

The reader is also briefly introduced to several topics of current interest, such as Toeplitz and subnormal operators, and commutator theory.

Several prominent features of the solutions deserve comment. First, the author displays a definite predilection for the soft, algebraic, discrete approach versus the hard, analytic, continuous one. Second, he eschews proofs which invoke a powerful but peripheral theorem, preferring the longer but more elementary approach. (For example, there is a careful avoidance of the Baire Category Theorem.) Finally, the solution to Problem 165 (If a contraction is similar to a unitary operator, must it be unitary?) is too clever by half. Mention should be made of the more pedestrian solution (modify one weight in the bilateral shift), which is at once, simple, constructive, and more useful.

In conclusion, the style of the text is breezy and both beginners and experts will find it a lot of fun to read. Both should encounter enough problems to puzzle over. As an added attraction, the preface contains the final score in the eigenvalue-proper value contest.

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Plateau's problem: an invitation to varifold geometry by F. J. Almgren, Jr. Mathematics Monographs Series, Benjamin, New York, 1966. 74 pp. \$7.00; paper.: \$2.95.

This short book is devoted to the task of giving the reader a digestible, but yet a rather penetrating account of a new and promising approach to the old and formidable Plateau's problem. In simple terms, the problem of Plateau asks for the existence and behavior of a surface of smallest area with prescribed boundary. The precise formulation of the problem depends upon the definitions that are adopted for "surface," "area," and "boundary," and this book describes a setting in the framework of geometric analysis which gives meaning to these terms, so that a solution can be found to a very general form of the problem. The setting is similar to the one that was employed by W. H. Fleming and H. Federer in their development of the theory of integral currents. The concept of surface in this approach assumes a role similar to that played by distributions in the theory of differential equations and the definition used for "surface" stems from the notion of *generalized surface* which was created by L. C. Young some twenty-five years ago. Thus, a k -dimensional surface (or in the terminology of this book, a k -dimensional *varifold*) is a particular kind of functional defined on the space of infinitely differentiable k -forms. A smooth surface can be regarded as a varifold

by identifying the surface with the functional obtained by integrating k -forms over the surface.

One of the most important features of the space of varifolds is a certain compactness property which makes it possible to obtain a varifold as a "weak" solution to Plateau's problem. While the local structure of such solutions is not yet completely known, the author states some theorems valid for two-dimensional varifolds in Euclidean three space, R^3 , that improve upon the results of T. Rado and J. Douglas concerning least area problems. Perhaps the most interesting is the one that combines the results of W. H. Fleming and E. R. Reifenberg and which states that if a boundary C is prescribed which is the union of a finite number of disjoint simple closed smooth curves in R^3 , then there is a two-dimensional varifold V which is a solution to this problem and has the property that the support of V minus the points of C is a two-dimensional real analytic manifold. The beauty of this solution comes from the fact that varifolds include all smooth surfaces and thus the solution V represents a surface of least area among all competing surfaces and not merely one of least area among those of prescribed topological type.

This book consists of four chapters, the last two of which describe varifolds and variational problems involving varifolds. In order to set the stage for this discussion, the first chapter gives an interesting expository account of least area phenomena and the second deals with the subject of rectifiable subsets of Euclidean space. A k -dimensional rectifiable set is one which can be approximated (in the sense of k -dimensional Hausdorff measure) by smooth k -manifolds. Rectifiable sets have become objects of fundamental importance to measure theoretic geometry since they are essential in describing the structure of sets of finite k -dimensional Hausdorff measure.

The material in this book is designed to be accessible to students who had a solid course in advanced calculus and it would serve nicely as a supplement to a course in the calculus of variations. The numerous revealing examples, which are accompanied by illustrations, will enable the reader to obtain a strong intuitive grasp of the many intricacies that are associated with Plateau's problem.

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Lie theory and special functions by Willard Miller, Jr. Mathematics in Science and Engineering, vol. 43, Academic Press, New York, 1968. xv+338 pp. \$16.50.

The principal aim of this book is to provide a group-theoretic interpretation of certain properties of the special functions of math-