SOME HOMOTOPY OF STUNTED COMPLEX PROJECTIVE SPACE

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Communicated by F. P. Peterson, August 15, 1967

1. **Introduction.** The 2-components of the stable homotopy groups $\pi_{2n+i}^{s}(CP/CP^{n-1})$ of stunted complex projective space are here tabulated, up to group extension, for $8 \leqq i \leqq 13$. For earlier work, including computation of these groups for $i \leqq 7$, see [8], [2], [7], [3], [4], and [5] as corrected by [6]. See [1] for odd components.

A result of Toda [8] relates these stable groups to the metastable homotopy groups of unitary groups as follows: Let $0 \leqq t < n$. Then $\pi_{2n+2t+1}^{s}(CP/CP^{n-1}) = \pi_{2n+2t+1}U(n)$, while there exists a commutative diagram with an exact row

$$\begin{array}{ccccccccc} 0 & \longrightarrow & Z & \longrightarrow & \pi_{2n+2t}^{s}(CP/CP^{n-1}) & \longrightarrow & \pi_{2n+2t}U(n) & \longrightarrow & 0 \\ & & \downarrow{\scriptstyle (n+t)!} & & \downarrow{\scriptstyle h} & & & & \\ & & Z & = & H_{2n+2t}(CP/CP^{n-1}) & & & & \end{array}$$

in which h is the Hurewicz homomorphism.

In view of Toda's formula the value of h is needed to deduce $\pi_{2n+2t}^{-}U(n)$. We include this data as (2.3) and give in (2.5) the order of the image of each element of the 2-component of $\pi_{2n+i}^{s}S^{2n}$ in $\pi_{2n+i}^{s}(CP/CP^{n-1})$.

Our basic method is the stable homotopy exact couple resulting from the standard cell filtration of CP/CP^{n-1}. By naturality, differentials in the resulting spectral sequence for CP/CP^{n-1} may be computed in the analogous spectral sequence for CP. The study in [6] of this sequence for CP is the basis of the calculation here; a more detailed description of the calculation will appear elsewhere.

2. **Results on homotopy groups.**

THEOREM 2.1. *The 2-component of the torsion of the stable homotopy group* $\pi_{2n+i}^{s}(CP/CP^{n-1})$, $8 \leqq i \leqq 13$, *is given by Table* 2.2.

In (2.2) nZ_2 denotes the direct sum of n copies of Z_2, while A ? B denotes a group satisfying an exact sequence $0 \rightarrow A \rightarrow A$? $B \rightarrow B \rightarrow 0$. Note that $Z_2 + Z_2$? $2Z_2$ denotes $Z_2 + A$, where $A = Z_2$? $2Z_2$, rather

TABLE 2.2

i \ $n(8)$	0	1	2	3	4	5	6	7
8	$3Z_2$	0	$2Z_2$	Z_4	$3Z_2$	0	$2Z_2$	Z_2
9 8(16)	$3Z_2 + Z_8$ $3Z_2 + Z_{16}$	Z_4 9(32) Z_8 25(32) Z_{16}	$Z_3 + Z_2 ? 2Z_2$ 10(32) $Z_2 + Z_2 ? (Z_2 + Z_4)$ 26(64) $Z_2 + Z_2 ? (Z_2 + Z_8)$ 58(64) $Z_2 + Z_{16} + Z_2 ? Z_2$	Z_8	$2Z_3 + Z_3 ? Z_4$	Z_2	$Z_2 + Z_2 ? Z_2$	Z_8 15(16) Z_{16}
10	Z_2	0	$2Z_2$	Z_2	Z_2	0	Z_2	Z_2
11 0(128) 8(32) 24(32)	$Z_8 ? Z_4$ $Z_8 + Z_4$ $Z_8 ? Z_8$ $Z_4 + Z_{16}$	$Z_4 ? 2Z_2$ 9(32) $Z_4 ? (Z_2 + Z_4)$ 25(64) $Z_4 ? (Z_2 + Z_8)$ 57(128) $Z_4 ? Z_2 + Z_{16}$ 121(128) $Z_4 ? Z_2 + Z_{32}$	$Z_2 ? Z_8$	$Z_2 + Z_8$	$Z_4 ? Z_2$	$Z_8 ? Z_2$	Z_{16} 22(32) Z_{32} 6(32) Z_{64}	$2Z_2 + Z_8$ 23(32) $2Z_2 + Z_{16}$ 39(64) $2Z_2 + Z_{32}$ 7(64) $2Z_2 + Z_{64}$
12	0	Z_2	Z_2	0	0	0	Z_2	0
13 8(32) 24(64) 56(128) 120(128)	$Z_4 ? 2Z_2$ $Z_4 ? (Z_4 + Z_2)$ $Z_4 ? (Z_8 + Z_2)$ $Z_4 ? Z_2 + Z_{16}$ $Z_4 ? Z_2 + Z_{32}$	$Z_2 ? Z_8$	$Z_2 + Z_8$	Z_8	$Z_8 ? Z_2$	Z_{16} 21(32) Z_{32} 5(32) Z_{64}	$2Z_2 + Z_8$ 22(32) $2Z_2 + Z_{16}$ 38(64) $2Z_2 + Z_{32}$ 6(64) $2Z_2 + Z_{64}$	$Z_8 ? Z_4$ 23(32) $Z_4 + Z_{16}$ 7(32) $Z_8 ? Z_8$

than $2Z_2$? $2Z_2$. The multiple entry for $i=9$ and $n\equiv 1(8)$ should be read as Z_4 except for $n\equiv 9(32)$ and $n\equiv 25(32)$.

Modulo torsion, $\pi^s_{2n+i}(CP/CP^{n-1})$ vanishes for i odd, but is infinite cyclic for i even.

THEOREM 2.3. *Let x_{n+k} generate $H_{2n+2k}(CP/CP^{n-1})$. Let $h_{n+k,k}x_{n+k}$ generate the image of the Hurewicz homomorphism $h\colon \pi^s_{2n+2k}(CP/CP^{n-1}) \to H_{2n+2k}(CP/CP^{n-1})$. Then, up to multiplication by an odd integer, for $1 \leqq k \leqq 6$ $h_{n+k,k}$ is given by Table 2.4, while $h_{n+7,7}=h_{n+7,6}$.*

TABLE 2.4.

$n(8)$ \ k	1	2	3	4	5	6
0	1	2	2	8	8 8(16) 4	16 8(16) 8
1	2	2	8	8 9(16) 4	16 9(32) 8 25(32) 4	16 9(32) 8 25(64) 4 57(128) 2 121(128) 1
2	1	8	4	8 10(16) 4	8 10(32) 4 26(64) 2 58(64) 1	128
3	2	4	4	8 11(32) 4 27(64) 2 59(64) 1	32	64
4	1	4	1	16	16	64
5	2	1	8	16	32	16
6	1	8	4	16	16	64 22(32) 32 6(32) 16
7	2	4	2	16	16 15(16) 8	32 23(32) 16 39(64) 8 7(64) 4

THEOREM 2.5. *Let β be in the 2-component of G_i, $0<i\leqq 13$. Let j: $S^{2n}\to CP/CP^{n-1}$ be the inclusion. Then the order of $j_*\beta\in\pi^s_{2n+i}\cdot(CP/CP^{n-1})$ is given by Table* 2.6.

Nomenclature for elements of G, the stable homotopy of spheres, is as in [9].

TABLE 2.6.

$n(8)$ \ β	η	η^2	ν	ν^2	σ	ϵ	$\bar{\nu}$	$\eta\sigma$	$\eta\epsilon$	$\eta\bar{\nu}$	$\eta^2\sigma$	μ	$\eta\mu$	ζ
0	2	2	8	2	16	2	2	2	2	2	2	2	2	8
													24(32)	4
1	0	0	2	2	8	0	0	0	0	0	0	0	0	2
				9(16)	16						25(32)	2	121(128)	4
2	2	2	2	0	8	2	2	2	2	0	2	2	2	2
				10(16)	16									
3	0	0	0	0	2	2	2	0	0	0	0	0	0	0
				11(32)	4									
				27(64)	8									
				59(64)	16									
4	2	2	4	2	8	2	2	2	2	2	2	2	2	4
5	0	0	4	2	4	0	0	0	0	0	0	0	0	4
6	2	2	2	0	2	2	2	2	2	0	2	2	2	0
													22(32)	4
													6(32)	8
7	0	0	0	0	2	0	0	0	0	0	0	0	0	0
											15(16)	2	39(64)	2
													7(64)	4

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SOME PROPERTIES OF DISTRIBUTIONS WHOSE PARTIAL DERIVATIVES ARE REPRESENTABLE BY INTEGRATION[1]

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Communicated August 21, 1967

We denote n dimensional Euclidean space by R^n and let H^m be m dimensional Hausdorff measure.

It is well known that distributions of the type described in the title may alternately be characterized as corresponding to H^n measurable real valued functions f with the following property: *There exists a sequence of infinitely differentiable real valued functions* f_j *on* R^n *such that*

$$\lim_{j\to\infty}\int_K |f_j - f|\, dH^n = 0 \quad \textit{and} \quad \liminf_{j\to\infty}\int_K \|Df_j\|\, dH^n < \infty$$

for every compact subset K *of* R^n. The class of such functions f is now widely regarded as the proper generalization to $n>1$ of the class of those functions on R which are H^1 equivalent to functions with finite total variation on every compact interval. However up to now there has been lacking an extension to $n>1$ of the basic classical results describing the continuity properties of functions with locally finite variation, namely that the set of points of discontinuity is countable and that one-sided limits exist everywhere. At first sight such an

[1] This work was supported in part by a research grant from the National Science Foundation.