RESEARCH PROBLEMS

2. Richard Bellman: *Some problems concerning convolutions.*

A. Let u, v, w be three functions defined for $t \geqq 0$ and connected by the relation $w(t) = \int_0^t u(s)v(t-s)ds$, commonly written $w = u * v$, the *convolution* of u and v. There are many questions connected with the determination of u, given v and w. Consider the more general class of problems:

(a) Let u, v, and w be given respectively on subsets S_1, S_2, and S_3 of the interval $[0, \infty)$. Under what assumptions, concerning S_1, S_2, and S_3 and the behaviors of u, v, and w on those sets, are the three functions uniquely determined by the convolution relation?

(b) Assuming that they are uniquely determined, what algorithms exist for evaluating the functions?

B. Consider the same problems for the case where

(a) $w_n = \sum_{k=0}^{n} u_k v_{n-k}$, $n = 0, 1, \cdots$,

(b) $w_n = \sum_{k/n} u_k v_{n/k}$, $n = 1, 2, \cdots$.

(Received October 12, 1964.)

3. D. Suryanarayana: *Research problems in theory of numbers.*

Problem A: Evaluate $\prod_{P\equiv 1(4)} (1-p^{-s})$ for $s>1$, where the product is taken over all primes p congruent to 1 modulo 4. In particular, what is the value of the product for $s=2$.

Problem B: Is there an odd integer n with the following properties:

(i) n is a square,

(ii) $(n, \sigma(n)) = 1$, where $\sigma(n)$ denotes the sum of all the positive divisors of n.

(iii) $\sigma(\sigma(n)) = 2n$.

Problem C: Let P_n denote the nth prime, where $P_1 = 2$ and let $s = s(n)$ be the positive integer such that $\prod_{\gamma=n}^{s-1} P_\gamma/(P_\gamma - 1) < 2 < \prod_{\gamma=n}^{s} P_\gamma/(P_\gamma - 1)$. Then show that $P_s > P_n^2$ for all $n \geqq 4$. (The relation $P_s > P_n^2$ can be verified to be true for $4 \leqq n \leqq 100$ with the help of the table given at the end of the paper entitled *Remarks on the number of factors of an odd perfect number* by Karl K. Norton published in the Acta Arithmetica **6** (1961), 365–374.) (Received October 30, 1964.)