RESEARCH PROBLEMS

1. W. R. Utz: *The equation $f'(x)=af(g(x))$.*

Determine conditions for the existence of a real function $f(x)$, not identically zero, satisfying $f'(x)=af(g(x))$ wherein a is a given real constant and $g(x)$ is a given real function. The prime denotes differentiation with respect to x. In general, the equation is not included in the theory of differential equations.

The equation $f^{(n)}(x)=f(x^{-1})$ has been solved by P. N. Sarma [1] and L. Silberstein [2].

More generally, it is only an exercise to determine analytic solutions, when they exist, of $f^{(n)}(x)=af(bx^s)$ when appropriate reals n, a, b, and s are given. For example, the functions $f(x)=A(\sin ax+\cos ax)$ satisfy $f'(x)=af(-x)$ and the functions $f(x)=A\cosh ax+B\sin ax$ satisfy $f''(x)=a^2f(-x)$. A and B are arbitrary real constants.

References

1. P. N. Sarma, *On the differential equation $f^{(n)}(x)=f(x^{-1})$*, Math. Student **10** (1942), 173–174.

2. L. Silberstein, *Solutions of the equation $f'(x)=f(x^{-1})$*, Philos. Mag. **30** (1940), 185–187.

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