tions appropriate for the regions with the coefficients in the expansions being the unknowns. Algebraic equations in $\mathfrak{N}$ for these coefficients are derived by equating the solutions for the two subregions and their normal derivatives on the interface between the regions.

The two regions can arise from functional as well as geometric considerations. Thus, consider the potential of a cylinder, say $\{0<r<1; 0<x<1\}$ where $u=0$ on $\{r=1; 0<x<1/2\}$ and $\partial u/\partial r=0$ on $\{r=1; 1/2<x<1\}$, and $u=1$ on $\{0<r<1; x=0\}$ and $\{0<r<1; x=1\}$. In this case the two regions would be $\{0<r<1; 0<x<1/2\}$ and $\{0<r<1; 1/2<x<1\}$.

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ON LOCAL DIFFEOMORPHISMS ABOUT AN ELEMENTARY FIXED POINT

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Let R^n be the real n-space with O as the origin. Let $\boldsymbol{G}$ be the group of the germs of C^∞ local diffeomorphisms about O as a fixed point. We say that T, $T'\in \boldsymbol{G}$ are equivalent if they are conjugate in the group $\boldsymbol{G}$. Denote by Θ the natural homomorphism from the group $\boldsymbol{G}$ onto the group $\mathfrak{G}$ of the ∞-jets at O. The fixed point O of $T\in \boldsymbol{G}$ will be said to be elementary if the Jacobian $J(T)$ has no eigenvalue of absolute value equal to 1.

Let $\boldsymbol{A}$ be the Lie algebra (over R) of the germs of C^∞ local vector fields about O and vanishing at O. We also use Θ to denote the natural homomorphism of the Lie algebra $\boldsymbol{A}$ onto the Lie algebra $\mathfrak{A}$ of the ∞-jets.

THEOREM 1. *Let the fixed point O of $U\in \boldsymbol{G}$ be elementary. Then U is equivalent to $T=\phi\eta$, ϕ, $\eta\in \boldsymbol{G}$, such that*

(a) *ϕ is a nonsingular semisimple (i.e., diagonalizable over the field of the complex numbers) linear transformation of R^n,*

(b) *$J(\eta)$ is equal to the identity mapping of R^n plus a nilpotent linear tranformation,*

(c) *$\phi\eta=\eta\phi$.*

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Analogous to the proof of the corresponding theorem in [1], the proof of this theorem is divided into three steps: In the first step, we show that ΘU is equivalent in $\mathfrak{G}$ to $\hat{T}=\hat{\phi}\hat{\eta}$, where $\hat{\phi}$, $\hat{\eta}\in\mathfrak{G}$ satisfy the conditions (a), (b), (c) of the theorem. Instead of directly seeking $\hat{\sigma}\in\mathfrak{G}$ such that $\hat{T}=\hat{\sigma}^{-1}\Theta U\hat{\sigma}$, we construct $\hat{Z}\in\mathfrak{A}$ such that $\hat{\sigma}=\operatorname{Exp}\hat{Z}$. The second step involves the construction of ϕ, $\eta\in \boldsymbol{G}$ with $\Theta\phi=\hat{\phi}$, $\Theta\eta=\hat{\eta}$ such that ϕ and η commute. In the third step, we show that U and $T=\phi\eta$ are equivalent. Here we use a modification of S. Sternberg's method for a more restricted situation [5].

Since, by construction, T possesses stable and unstable manifolds, we conclude that U also possesses stable and unstable manifolds (see [4]).

THEOREM 2. *Let the fixed point O of U, $U'\in \boldsymbol{G}$ be elementary. Then U and U' are equivalent in $\boldsymbol{G}$ if and only if ΘU and $\Theta U'$ are equivalent in $\mathfrak{G}$.*

PROOF. Necessity of the above theorem is clear. For sufficiency, we observe that U' is equivalent to U'' such that $\Theta U=\Theta U''$. Since the construction of $T=\phi\eta$ depends only on ΘU, the theorem follows immediately.

Theorem 2 is a generalization of Sternberg's theorem [5] in the sense that we have removed the condition of semisimplicity of $J(T)$, which was assumed in Sternberg's proof.

THEOREM 3. *Let the fixed point O of $U\in \boldsymbol{G}$ be elementary. Then*

$$U = \operatorname{Exp} X$$

for some $X\in \boldsymbol{A}$ if and only if $\Theta U=\operatorname{Exp}\hat{X}$ for some $\hat{X}\in\mathfrak{A}$.

The above theorem is a consequence of Theorem 2 and an analogous theorem for vector fields given in [1].

THEOREM 4. *Let the fixed point O of $U\in \boldsymbol{G}$ be elementary. Then there exists a positive integer α depending only on the eigenvalues of $J(U)$ such that*

$$U^{\alpha} = \operatorname{Exp} X$$

for some $X\in \boldsymbol{A}$.

For any $\hat{T}\in\mathfrak{G}$, we say that $\hat{\phi}\in\mathfrak{G}$ is the semisimple part of $\hat{T}$ if
(a) $\hat{\phi}$ is equivalent to the jet of a linear transformation of R^n,
(b) $J(\hat{\phi})$ is the semisimple part of $J(\hat{T})$,
(c) $\hat{T}\hat{\phi}=\hat{\phi}\hat{T}$.
For any $\hat{T}\in \boldsymbol{G}$, the semisimple part $\hat{\phi}$ exists and is unique.

THEOREM 5. *Let $\hat{\phi}$ be the semisimple part of $\hat{T}$. Then*

$$\hat{T} = \operatorname{Exp} \hat{X}$$

for some $\hat{X} \in \mathfrak{A}$ if and only if there exists $\hat{S} \in \mathfrak{A}$ such that

(a) $\hat{\phi} = \operatorname{Exp} \hat{S}$,

(b) $(\operatorname{Adj} \hat{T})\hat{S} = \hat{S}$, *i.e., $\hat{T}$ leaves $\hat{S}$ invariant.*

From the above theorem follows the formal analogy of Theorem 4 (due to Lewis [3]) and therefore Theorem 4.

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