## BOOK REVIEWS

*A primer of real functions.* By R. P. Boas, Jr. Carus Monograph No. 13. Wiley, New York, 1960. 13+189 pp. $4.00.

Long before this book was published Professor Boas told me of his plans to write a new Carus monograph on a less specialized topic than those presented in the last few volumes. He had already decided to write a first introduction to real variable theory and in order to have a guiding line he planned to include everything which is necessary to formulate and prove the following proposition:

Suppose the continuous real valued function $f$ of a real variable $x$ has derivatives of all orders everywhere and for each $x$ there is an order $n(x)$ such that the $n(x)$th derivative of $f$ vanishes at $x$. Then $f$ is a polynomial function.

The material is divided into two chapters entitled "Sets" and "Functions", respectively. The text begins with the introduction of naive set theoretic notions such as unions, intersections, complements, countability and one-to-one mappings. This is followed by the definition of a metric space and a discussion of the elementary topological concepts associated with a metric. This includes open and closed sets, intersections, boundary and cluster points, the notion of connectedness, dense and nowhere dense sets, separability, compactness and convergence. Several useful results are stated as exercises which abound not only in the beginning, but in almost every part of the book. The lively discussion touches upon several applications of the set theoretic and topological concepts; these range from the existence of transcendental numbers to lion hunting. It is good to see several fundamental results which are often ignored in the more systematic general topology texts. For instance, the structure of the open sets on the real line is explicitly formulated. In the preface the reader is advised to skip the harder parts and it seems that after reading the first eight paragraphs the beginner should switch to the second chapter and get acquainted with the concepts of continuity, boundedness and derivatives before reading §§9, 10, and 11 in the first chapter. In these last sections first and second category sets are introduced and Baire's theorem is proved. Among the applications we find a proof of the proposition mentioned in the beginning of this review and the proof of the existence of nowhere monotonic functions and of nowhere differentiable functions of a real variable.

A great deal of valuable material is presented in the second chapter. The effects of the author's broad knowledge and aesthetic sense can be noticed already in the beginning where continuity is discussed: It is explicitly stated that a continuous one-to-one map from a compact space to a Hausdorff space is necessarily a homeomorphism. The applications of the intermediate value theorem include the horizontal chord theorem, Borsuk's antipodal theorem and other tasty topics such as how to cut pancakes, sandwiches, and what Brouwer once said about the continuous stirring of a cup of coffee. It is a pleasure to see Pólya's theorem on the convergence of monotonic functions, proof included. In §18 it is proved that the pointwise limit of continuous functions of a real variable is continuous everywhere except possibly on a set of first category. The next section is devoted to uniform continuity and approximation on compact intervals. Weierstrass' approximation theorem is proved by Landau's method and the theorem about moments of continuous functions is given as an application. In spite of the limited space Boas manages to give clear and detailed account on the functional equation $f(x+y)=f(x)+f(y)$, on Dini derivatives, one sided derivatives, finite and infinite derivatives. The discussion includes monotonicity criteria, and the theorem on the upper and lower bounds of Dini derivatives. Saks' theorem on the three alternatives is stated without proof. The last sections deal with monotonic, convex and infinitely differentiable functions, respectively. Here the outstanding feature is a reproduction of a proof by Frédéric Riesz of Lebesgue's theorem which states that a monotonic function of a real variable has a finite derivative almost everywhere. The applications are well chosen: The first is Fubini's theorem on term by term differentiation of series of monotonic functions and the second is the useful lemma stating that if a set $S$ lies on the line then almost all points of $S$ are density points of $S$. It is proved that a continuous convex function of a real variable has finite derivative in all but countably many points. Hölder's and Minkowski's inequalities are derived, but Jensen's name is never mentioned. Near the end of the book we find historical remarks, references and sixteen pages devoted to the solution of the exercises. The index is very adequate.

There are a few instances where the text could be improved. It is particularly disturbing that the concept of a topological space is not even mentioned. Instead, on page 2, we find the following sentence: "A *space* is a set that is being thought of as a universe from which sets can be extracted." In spite of Dieudonné's long stay at Northwestern, the present book offers no help in understanding the difference between topological and uniform notions. Only two alternatives are offered: The first is the sentence quoted above and the second is a metric space with a fixed metric. Since in finite dimensional Euclidean spaces all sets of finite diameter happen to be totally bounded the inexperienced are tempted to see something of importance in sets having finite diameter. On page 22 we find: "A set is called *bounded* if it is contained in some neighborhood." And on page 32: "A set $E$ is *nowhere dense* if its closure contains no neighborhood." (Neighborhoods are introduced on page 22.) These unfortunate aspects are heavily outweighed by the wealth of useful information and in case of a second edition could be easily eliminated.

The book is clearly written by a man who knows mathematics, has something to say and is able to communicate with others. Undergraduates and promising high school students could profit a great deal by reading it.

I. S. GÁL

*Commutative algebra*, Vol. II. By O. Zariski and P. Samuel. Van Nostrand, New York, 1960. 10+414 pp. $7.75.

In this, the second volume of their treatise on commutative algebra, the authors have presented the basic facts and concepts of commutative ring theory essential to the practice of algebraic geometry as epitomized by the authors' work in the subject. The main topics covered are valuation theory, ideal theory in polynomial and power series rings, and local algebra. The volume ends with a series of seven appendices, some of which are devoted to generalizations and alternate routes to results given in the book proper and some of which are devoted to the introduction of new concepts. The prime example of the latter is the notion of a complete ideal in a noetherian domain (in the sense of integral closure) and Zariski's impressive theory of such ideals in regular local rings of dimension two.

As far as expositional style is concerned, the authors have fortunately seen fit to maintain the same leisurely manner established in their first volume, even though the material presented here is more technical and specialized. Many examples are given and they have not hestiated to give different proofs for the same theorems in those cases where this is appropriate due to the unique features of the various proofs. Also their practice of presenting the same theorems or theories in varying degrees of generality gives a sense of continuity and tentativeness to their development of the material which strongly encourages the reader to try his hand at seeing if he can push things further. It is a pity that there are not more books written by masters of their subject who are as successful as Zariski and Samuel in resist-