amusing, therefore, to view Appendix I as a scale against which Kac's achievements can be measured. When Uhlenbeck mentions the problem of finding "appropriate laws for the approach to equilibrium" or of relating the classical Liouville approach for the description of a gas to the master equation, the reader will find comfort in a more thorough discussion of these points in the "ring" model and the Kac monatomic gas model of Chapter III. On the other hand, when Uhlenbeck mentions the Bogoliubov approach to the description of a gas, the corresponding feeling of comfort is not to be found. Uhlenbeck's discussion is good, but quite intuitive (rather than rigorous) and a bit brief at times. To the reviewer, the real issue is to define the problems in a more reasonable form. Because of this problem of definition, Appendix I may well give the reader a much deeper appreciation for the accomplishments of Kac in Chapter III. At the same time the reader will discover that Kac and Uhlenbeck talk a somewhat different language. In spite of their many years of association, a large gap still exists between their viewpoints.

An excellent set of notes and bibliography has been included at the end of the book. Readers interested in the history of the problems discussed will find these notes add a certain glamor to the various sections.

There are several groups to whom this book is highly recommended. All persons interested in probability theory will find the models and examples presented well worth their attention. The physicist with prior knowledge of probability theory, who is interested in applications, will find this an excellent exposition of the methods and techniques involved. In summary, for all those interested in probability and related topics in physical science, the book, will serve as a concise and elegant summary of the author's own investigations into this important borderline area.

GLEN BAXTER

Homotopy theory. By Sze-Tsen Hu. New York and London, Academic Press, 1959. xiii+347 pp. $11.00.

Although homotopy theory has been rather intensively studied for 25 years and is today one of the principal branches of algebraic topology, this is the first textbook on the subject. (A possible exception to this statement is the Cambridge Tract by P. J. Hilton entitled *An introduction to homotopy theory*; however, this booklet is only about 140 pages long and is principally concerned with certain special topics.) Since this book by Hu is the only text in this large field, its acquisition is a "must" for any mathematical library having any pretensions to completeness. For the graduate student or mature mathematician who is not a topologist and wishes to learn something about homotopy theory, it should be the primary reference.

Writing a satisfactory book on homotopy theory is an extremely difficult matter, considering the present state of the subject. Any expert in the field who tries to do so is inevitably laying himself wide open to criticism by all the other experts. It is to the author's credit that he frankly recognizes these difficulties, as indicated by the following statement from the preface: "This book is by no means designed to be an exhaustive treatment of its subject; for example, the recent celebrated contribution of M. M. Postnikov is not included. Besides, homotopy theory is advancing so rapidly that any treatment of this subject becomes obsolete within a few years." The more modest nature of the author's aims is precisely indicated by the following paragraph from the preface:

"The present book is designed to guide a reader, who might be a beginning student or a newcomer to this branch of mathematics and who has a little knowledge of elementary algebraic topology, through the basic principles of homotopy theory. The author has aimed to provide the reader with sufficient detail for him to understand the fundamental ideas and master the elementary techniques so that he may be able to study the more advanced and more complicated results directly from the original papers."

We will now discuss the contents of the book in some detail, chapter by chapter.

Chapter I, entitled "Main problem and preliminary notions," and Chapter II, "Some special cases of the main problems," give background material for the rest of the text. In addition to introducing some basic concepts which are used throughout the text, Chapter I contains generalities on the problems of extending and classifying continuous maps; Chapter II takes up the first nontrivial cases of the homotopy classification and extension problem, namely, the case of maps of a 1-sphere into a space (the fundamental group) and the classical Hopf classification and extension theorems for maps into an n-sphere. It is the author's contention that "many theorems of topology and most of its applications in other fields of mathematics are solutions of special cases of the extension problem."

Chapter III is called "Fiber spaces." It is generally conceded that fiber spaces are essential to the study of homotopy theory; in the treatment of the subject by Hu, their definition precedes that of the homotopy groups. The author uses the definition of fiber spaces based on the covering homotopy property (due to Serre), although some of the older definitions are also mentioned and compared. Covering spaces are defined and shown to be fiber spaces with discrete fiber. Certain spaces of paths and function spaces are shown to be fiber spaces.

Chapter IV, "Homotopy groups," gives the basic definitions and properties of the absolute and relative homotopy groups, homomorphisms induced by a continuous map, exact homotopy sequences, effect of change of base point, etc. Also included is a system of axioms which characterizes homotopy groups uniquely. The effective calculation of some homotopy groups first occurs in Chapter V. The main topics discussed in Chapter V are the Hurewicz isomorphism theorem and the exact homotopy sequence of a fiber space. Freudenthal's suspension is also introduced.

Chapter VI gives an account of the standard theory of obstructions to extensions and homotopies of continuous maps, while Chapter VII is concerned with the Borsuk-Spanier cohomotopy groups. The subject matter of this latter chapter does not seem to be of as fundamental a nature as that of most of the preceding six chapters and is undoubtedly in a more fluid state. More recent research especially that of Spanier himself and J. H. C. Whitehead, has resulted in a changed point of view toward the cohomotopy groups.

The last four chapters are of a distinctly more advanced nature than the first seven chapters. Their purpose is to introduce the reader to some of the more recent developments in homotopy theory, especially those based on the use of the spectral sequence of a fiber space (as introduced by J.-P. Serre in his thesis in 1951). Chapter VIII, entitled "Exact couples and spectral sequences," develops the purely algebraic part of the theory. Chapter IX, "The spectral sequence of a fiber space," develops the theory indicated in the title using cubical singular homology à la Serre. A variety of applications, such as the Gysin sequence, the Wang sequence, etc., is given. Chapter X, "Classes of Abelian groups," is based on a well-known paper of Serre in the Annals of Mathematics for 1953. The last chapter, entitled "Homotopy groups of spheres," describes the computation of the groups $\pi_{n+r}(S^n)$ for $r \leqq 4$ using the methods developed in the three preceding chapters; here again most of the methods and results are due originally to Serre. At the end of the chapter there is a table giving (without proof) the structure of the groups $\pi_{n+r}(S^n)$ for $5 \leqq r \leqq 8$.

At the end of the book there is a bibliography of slightly more than four pages. In order to forestall criticism of the inadequacy of the bibliography, the author makes the following statement in the preface: "The bibliography at the end of this book has been reduced to the minimum essential to the text and the exercises. References to this bibliography are included for the convenience of the reader so that he can find more details concerning the material; the references are not intended to be a historical record of mathematical discovery. Frequently, expository articles are preferred to the earlier original papers."

At the end of each chapter there is a list of exercises. Some of these lists are rather long. Probably the main criticism that one can aim at them is that some are not really exercises, but are summaries of the main results of certain papers together with a suggestion that the reader study the paper in question. For example, an exercise on page 258 asks the reader to "Reproduce the definition of the Steenrod square operations

$$Sq^i\colon H^n(X, A, Z_2) \to H^{n+i}(X, A, Z_2), \qquad i \geqq 0,$$

and prove the following properties of these operations." Then follows a list of seven properties of the Steenrod square together with references to papers of Steenrod and Cartan. No hints or explanatory comments are given. As another example, on page 139 the reader is asked to prove several properties of the Whithead product as an exercise, among which is the Jacobi identity. No hints or references are given. Apparently the author has forgotten that the Jacobi identity was for many years an unproven conjecture; finally proofs were discovered simultaneously by various people, and at least four papers were published giving different proofs, none of which was trivial or obvious.

Clearly it would have been preferable to confine the exercises to problems which one might hope a fair percentage of the intended readers could do by themselves with a reasonable amount of ingenuity and/or hard work. Examples such as those just mentioned might better have been included in the body of the text without proof but with an appropriate reference. Alternatively, the author might have included a "Guide to the Literature" similar to that given by I. Kaplansky at the end of his booklet entitled *Infinite Abelian groups.*

One comment about the style of writing seems in order. As mentioned above, in a statement in the preface the author disclaims any attempt to give an exhaustive treatment of his subject. For one who looks at the contents of the book from a global point of view, there is no doubt that this disclaimer is justified. However, if one examines the treatment of some of the topics from a strictly local point of view, one feels that the author did not try to resist the impulse to be exhaustive. A couple of examples will illustrate this point. In Chapter

III, after defining a fiber space by means of the covering homotopy property for maps of polyhedra, the author states and proves a theorem to the effect that fiber spaces can be characterized by any one of five equivalent properties. Certainly most experts in the field have never needed to consider these five equivalent properties; to the beginner in the subject, they are of even less importance. If the author felt the absolute necessity of including this result, it would have been preferable to indicate by some device that it was of secondary importance. For instance, it could have been placed in an appendix at the end of the chapter, or stated without proof in a footnote, or set in finer type than the rest of the text. Then the unwary reader would have known that he could skip this result and proceed directly to something more fundamental. Other examples of material which were apparently included for the sake of completeness are §6 of Chapter III entitled "Algebraically trivial maps $X \rightarrow S^2$" and §5 of Chapter V entitled "Direct sum theorems." In both these cases, the material is simple enough so that the reader could work out the details as an exercise if he were given a few hints; moreover, this material is not of such importance that its omission would prevent the reader from proceeding directly to the more complicated parts of the subject.

In a subject as complicated as Homotopy Theory, it is important that the reader be guided through the fundamentals of the subject as quickly as possible so he can get on to some of the deeper results and the more interesting current research. If the exposition of the basic aspects of the subject is cluttered up with a great many details for the sake of completeness, the reader is likely to become discouraged and give up. In the opinion of the reviewer, this is likely to prove to be the greatest weakness of Hu's book.

Continuing in the same vein, it is unfortunate that the author did not include a "Leitfaden" such as is given by Van der Waerden at the beginning of his famous text on algebra to show the logical dependence of the various chapters on each other. For example, most of the material in the last four chapters is independent of Chapters VI and VII; however, there is no easy way for the reader who is mainly interested in the material of the last chapter on homotopy groups of spheres to find this out for himself.

In spite of these criticisms, it must be admitted that the author has done a tremendous piece of work in writing this book. He has succeeded in doing what no other living topologist has even really seriously attempted.

W. S. MASSEY