kernels and the theory of linear integral equations of the first kind.

The book serves as an excellent introduction to the theory of integral equations. It is admirably organized and concisely written. No applications are given, except in the first chapter, where the connection with differential equations is sketched. However, this tract should prove a useful reference work for all who employ the theory in such applications.

JOANNE ELLIOTT

Advanced complex calculus. By Kenneth S. Miller. New York, Harper and Brothers, 1960. 8+240 pages. $5.75.

This book is intended as an introduction to complex variable theory on a level suitable for juniors, seniors, and beginning graduate students. There are nine chapters, whose headings follow: 1. Numbers and Convergence; 2. Topological Preliminaries; 3. Functions of a Complex Variable; 4. Contour Integrals; 5. Sequences and Series; 6. The Calculus of Residues; 7. Some Properties of Analytic Functions; 8. Conformal Mapping; and 9. The Method of Laplace Integrals.

The book has two nonroutine features: (1) The last chapter, an introduction to a differential equations topic, is included to give an application of contour integration; (2) A discussion of "Riemann axes," a real variable analog of Riemann surfaces, is given to serve as a motivation for the use of these surfaces in the study of multivalued analytic functions.

The author asserts in his preface that he pays "careful attention . . . to multi-valued functions." But his discussion of "Riemann axes" and of Riemann surfaces remains on a rather vague level. Nothing, for example, is said about whether branch points do or do not belong to such a surface. Also, the domains of single-valued branches of logarithms and powers are rarely mentioned. Indeed, the author's definition of *function* is hardly conducive to clarity: a function in the book is a "rule" which "associates with every point z in $\mathfrak{C}$ [a set of points in the plane] at least one point w."

The impact of this book on the student will be the formation of the opinion that complex variable theory is merely a jungle of theorems, many of which lead nowhere in the further development of the theory.

As an example, §7.5 is headed "The Maximum Modulus Theorem." In addition to a statement of this theorem, the section also contains Rouché's theorem, a uniqueness theorem for analytic functions, the open mapping theorem, and an inverse function theorem. After the proof of the maximum principle, which, incidentally, is defective, that principle is not mentioned again in the text, except that two of the problems at the end of Chapter 7 refer to it. One of these asks the student to prove the minimum principle and to lighten one of the hypotheses of the original theorem, and the other calls for a proof of Schwarz's lemma—without a suggestion that the function $f(z)/z$ should be considered or that this theorem depends on the maximum principle. (Also, the name of the discoverer is not mentioned with this problem.)

Indeed, this book suffers as a text because of its poor selection of exercises. These appear in sets at the ends of the chapters. They range from the rather trivial (e.g. Ex. 5.2) through problems that have no particular complex variable flavor (as, for instance, Ex. 5.1 and Ex. 5.3) to some that seem exceedingly difficult without suitable hints. Such hints, however, are not furnished. (Striking examples of this kind, in addition to the exercise on Schwarz's lemma, are Ex. 1.5 on the Schwarz inequality, Ex. 4.7 on the Poisson integral, and Ex. 7.13 on an arbitrary Jordan region being the natural domain of existence of a holomorphic function.) Equally serious is the fact that a number of opportunities to provide exercises have been missed. For instance, in §7.6 it is mentioned without proof (or indication that a proof is needed) that the unit circle is the natural boundary of $\Sigma z^{n!}$.

It seems surprising, in view of the claim in the preface that "the only important theorem that we do not prove is the Jordan separation theorem . . . " (p. vii), that Mr. Miller states, after mentioning but not proving the Riemann mapping theorem: " . . . The proof does not indicate how to construct such a [mapping] function, that is, it is a pure existence proof . . . " (p. 218). As a matter of fact, no real definition of a conformal map is given. The author wavers between "$f'(z_0)\neq 0$" (§2.11, §8.1) and "angles at z_0 are preserved" (§8.2) as the crucial requirement. Univalence does not seem to be demanded of a conformal map in this text.

In deviation from common usage, the author requires Jordan arcs and curves to be rectifiable as well as simple, and he defines connectedness in terms of arcwise connectedness in open sets (p. 24). However, in Exercise 2.11 he asks for the proof of the connectedness of a certain non-open set. It might also be mentioned that the fundamental role played by sets of points that are both open and connected is not brought out at all. ("Region" is not defined; when the term is used, it seems merely to mean "plane set of points.")

One final comment relates to an aspect of the organization of the text. The author has tried to keep the first five chapters parallel to those in his earlier book, *Advanced real calculus* (Harper and Brothers, 1957), and there are numerous references to that volume in the present one. This reviewer found disquieting the necessity for frequent consultation of other works that will be needed by a student to fill the gaps in *Advanced complex calculus*.

ERNEST C. SCHLESINGER

Individual choice behavior, a theoretical analysis. By R. Duncan Luce. New York, Wiley, 1959. 12+153 pp. $5.95.

This book belongs to that small but increasing set of teatises in which competent mathematicians apply their skill to problems in the behavioral sciences. Luce's basic concept is the probability that a subject chooses an alternative belonging to a specified subset of the set of alternatives presented. His primary interest is in the consequences of an axiom concerning these probabilities. Results are applied to psycho-physics, learning theory, and utility theory.

Excellent features of the book are the careful statement of assumptions, derivations, and results, the plentiful and good examples, the attention to interpretation, the candid accounts of the history and significance of problems, the mention of unsolved problems, and the attention to testing the theory. As an example of the last named, Luce is not satisfied to note that his utility model is capable of experimental verification "in principle." He points out that the indicated experimental study would be impractical and then derives a result that can be tested.

The book has significant implications for the mathematics curriculum. The reader is faced with only the most modest demands on his algebraic and computational skill. On the other hand, he is required to have considerable sophistication and appreciation of axiomatics and the ability to keep in mind various assumptions and their interrelations. Indeed the skills required are most similar to those that are developed in undergraduate courses in foundations of mathematics or in elementary courses of the so-called "modern" type. Classical analysis plays almost no role.

There are a few very minor blemishes to the generally excellent exposition. Quantification in the mathematical sections is rather clumsy. For example, Luce writes, in the statement of his major axioms and in many other places, the hypothesis "if $P(x, y) \neq 0, 1$, for all $x, y \in T$." This formulation is unfortunate on two counts: