## BOOK REVIEW

*Algebraische Gleichungen mit reellen oder komplexen Koeffizienten.* By Wilhelm Specht. Enzyklopädie der Mathematischen Wissenschaften, Band I, 1, Heft 3, Teil II. Stuttgart, B. G. Teubner Verlagsgesellschaft, 1958. 80 pp. DM 19.

This section of the second edition of the Enzykopädie seems to be a replacement of the article by C. Runge in the first edition. Entitled *Separation und Approximation der Wurzeln,* Runge's article dealt in part with the theorem of Descartes, Budan-Fourier and Sturm on the number of zeros of a real polynomial on a real interval and with two extensions of the Sturm theorem to the complex domain. But, since Runge's article was printed in 1899 and since most of the material reported in Specht's article was discovered during the present century, there is not too much overlapping between Runge's and Specht's articles.

Though entitled *Algebraic equations,* Specht's article (as it is explained in its introduction) deals with material usually called the "analytic theory of polynomials" or the "geometry of the zeros." As such, it overlaps considerably more the two books:

(I) J. Dieudonné, *La thëorie analytique des polynomes,* Memor. Sci. Math. Fasc. 93 (1938).

(II) M. Marden, *The geometry of the zeros of a polynomial in a complex variable,* American Mathematical Society, Mathematical Surveys, no. 3, 1949.

In size (76 pages) it is similar to (I) (70 pages) and like (I), it states results generally without proof and arranges them primarily in an order convenient for reference purposes. This contrasts with (II) which has 183 pages and so attempts to prove most of the results arranging them in an order determined by the methods employed.

While Specht's article reports most of the results stated earlier in (I) and (II), it does have some features which were omitted or were not given as much emphasis in (I) or (II). For instance, it tells about various proofs of the fundamental theorem of algebra and treats the continuity of the zeros at greater length than does (II). Important theorems on symmetric functions of the zeros, quadratic and hermitian forms and Kronecker's method of characteristics are reviewed. Also included are theorems concerning the characteristic roots of matrices, though unfortunately the results due to A. Brauer and Parodi are omitted. Likewise included are theorems on the zeros of linear combinations of orthogonal polynomials; reference, however, is omitted to earlier theorems by Laguerre, Obrechkoff and Marden in the real case. In addition, this article lists a number of theorems discovered since the publication of (I) and (II), including certain important ones due to Specht himself.

On the other hand, Specht's article is open to criticism for certain omissions. First, it omits reference to Pólya-Szegö's *Aufgaben und Lehrsätze aus der Analysis*, Berlin, 1925, which over the past 34 years has been one of the most reliable reference sources on polynomials and allied topics. Secondly, the theorem that "all the zeros of the derivative $f'(z)$ of a polynomial $f(z)$ lie in the convex hull of the zeros of $f(z)$" is named the "Gauss-Lucas Theorem" whereas in reality this theorem is due to Lucas alone. Gauss' theorem was that the zeros of $f'(z)$ are positions of equilibrium in the field due to particles at the zeros of $f(z)$ attracting according to the inverse distance law. Thirdly, in giving on page 31 the references for the Eneström-Kakeya theorem, Specht's article gives 1912 as the date of Kakeya's article but 1920 as that of Eneström's article, without noting that Eneström's 1920 article was a translation of an article published in Swedish in 1893 by Eneström. This information is, of course, needed to establish Eneström's priority on the theorem. Fourthly, in stating on pages 46–47 the Schur-Cohn results, it fails to mention the more compact form of these results as given in (II) and the simpler proof and generalizations of these results published by Bonsall and Marden in 1952 and 1954. Fifthly, it erroneously credits to E. Frank [Bull. Amer. Math. Soc. vol. 52 (1946) pp. 144–157; 890–898] the expressions on page 51 for the number $N^+$ of zeros in the right half plane, which expressions are stated and proved in (II), pages 139–141. These expressions are not given in the papers by Frank or Bilharz but might have been deduced from certain details in those papers.

However, none of these criticisms seriously detracts from the importance of Specht's article as a well-organized survey of the analytic theory of polynomials. It should prove to be a valuable up-to-date reference source to all who work regularly or even occasionally in this field.

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