

The scope of the book will become apparent from a brief survey of its contents. After treating the basic notions like convex cover, supporting and separating planes, convex polyhedra (called polytopes), the tract turns in Chapter 2 to Helly's and Carathéodory's theorems, their interrelation and some generalizations. Convex functions, in particular distance and supporting functions follow (Chapter 3). Then distance for convex sets, Blaschke's Selection Theorem and the approximation of a convex set by polyhedra and regular convex sets are discussed. Chapter 5 deals with linear and concave families of convex sets, mixed volumes, Steiner's symmetrization, the Brunn-Minkowski theorem (Brunn is spelled as Brünn throughout), and Minkowski's inequalities. The more general inequalities of Alexandrov-Fenchel are stated without proof.

In Chapter 6 we find some extremal problems like the isoperimetric problem, and the relations between the inradius, circumradius and the diameter, also Besicovitch's result on the asymmetry of plane convex sets. Sets of constant width are the topic of the concluding chapter.

The book will well serve its purpose of providing an introduction to the field for "those starting research and for those working on other topics who feel the need to use and understand convexity."

HERBERT BUSEMANN

The numerical solution of two-point boundary problems in ordinary differential equations. By L. Fox. New York, Oxford University Press, 1957. 11 + 371 pp. \$9.60.

If insistence on deductive reasoning is one of the characteristics of mathematics, the numerical solution of mathematical problems is not a branch of pure mathematics. In its methods and spirit it is more closely related to the applied sciences, in that incomplete induction based on experimental evidence is the ultimate criterion, even though a good measure of theoretical analysis is indispensable for guidance and interpretation. It is true that there are numerical procedures whose theory is so well understood that a result can be obtained which is safely embedded between double numerical inequalities. However, at this time, and for the foreseeable future, the number of practically important problems in this comfortable class is, and will remain, depressingly small. A good specialist in the art of computation should therefore be able to resist the mathematician in him, who might lure him into the ideal realm of pure analysis, away from his concrete problems, without, on the other hand, losing the incentive of availing himself of all the mathematics, highbrow or lowbrow,

that can be put in the service of computation.

The author of this book is steering a safe course between these two dangerous rocks, somewhat closer, perhaps, to the second than to the first. He is steeped in the British tradition of Numerical Analysis, which was developed before the rise of high-speed computing machines. Most of the content of the book is applicable to machine computation as well, but, on the whole, more reliance is put on the facility of the human computer for quick scanning and flexible independent judgments than on speed of operation.

The mode of presentation is so careful and leisurely that any computer with a bachelor's degree should find it easy reading. Every variant of procedure is illustrated by numerical examples worked out in great detail. Also, many of the mathematical tools, such as the necessary facts about finite differences, the solution of linear algebraic systems and eigenvalue theory are fully explained, if not always proved. These features account for the surprising length of a book devoted to a rather special subject.

A boundary value problem for differential equations can be solved either by replacing the full problem by a finite difference, i.e. a matrix, problem or by solving a suitable sequence of initial value problems at one point until the particular solution that also satisfies the conditions at the other end point has been approximated with satisfactory accuracy. Both methods are explained, but much more space is devoted to the former.

The author believes that the mesh length should always be kept fairly large, and that the truncation error should be reduced by employing refined difference approximations. In order to cope with the higher difference terms, he recommends that the first order approximating equation be solved first, and that the higher correction terms be then introduced and taken care of by some iterative scheme.

After two introductory chapters on *Finite differences* and *The solution of algebraic equations*, the author turns to second order boundary value problems. Surprisingly, the author considers the solution of first order equations, which are taken up next, a more subtle matter than the second order problems. The new difficulties stem from the fact that the first approximating difference equations on which the calculations are based involve central differences over two mesh lengths, and are therefore of second order. The extraneous solutions of the difference equation introduced in this manner may affect the stability of the procedure. Various techniques for controlling these unstable fluctuations are described in the book.

Next, differential equations of higher order are discussed, as well

as systems of equations. After that, there is a long chapter on eigenvalue problems. The presentation becomes more condensed in the following chapter devoted to *Initial value techniques for boundary-value problems*.

A special chapter deals with the accuracy of procedures introduced previously. In view of the difficulty of this subject the conclusions here are largely qualitative statements supported by many numerical examples. In the final chapter several topics are touched upon very briefly, such as discontinuous coefficients, Richardson's deferred approach to the limit, collocation, nonlinear eigenvalue problems, etc.

Inasmuch as computation is an art rather than a science it cannot be taught systematically. This book tries to transmit some of the skill, flexibility and flair which a good computer needs in addition to a knowledge of the standard methods. Such a mode of presentation tends to make a book somewhat repetitious and a little exhausting for consecutive reading. This is unavoidable and is not meant as a criticism. Once the reader has become familiar with the basic techniques by studying, say, the first 90 pages, he will find the rest of the book an extremely valuable reference for advice and help, whenever he has to solve boundary value problems for ordinary differential equations.

WOLFGANG WASOW

Problems in Euclidean Space: Application of convexity. By H. G. Eggleston, Pergamon Press, New York 1957. 8+165 pp. \$6.50.

The book is grouped around ten principal, essentially unrelated problems. The author motivates collecting these problems in a book by their implicit or explicit connection with convexity. Actually, there is a much stronger bond between them, namely the spirit of the arguments, which is that of Besicovitch: the problems are special, the proofs require highly ingenious and intricate arguments, but use only the most elementary tools, euclidean geometry and trigonometry besides some simple facts on point sets and measure in E^2 or E^3 . Most of the work is due to the author, some to others, in particular to Besicovitch.

The first problem concerns the characterization of a nonvoid open set in E^2 which is the intersection of a decreasing sequence of open connected sets. The second is Ulam's problem, whether a homeomorphism $(x, y) \rightarrow (x', y')$ of E^2 , or of a square in E^2 , on itself can be approximated by homeomorphisms in which only one variable is changed. Problem 3 deals with sets E of finite linear Hausdorff measure. Denote by E_α the projection of E on a line with direction α