ABSTRACTS OF PAPERS

The abstracts below are abstracts of papers presented by title at the October Meeting in New York and the November meeting in Evanston. Abstracts of papers presented in person at these meetings will be included in the reports of the meetings which will be published in the January issue of this BULLETIN.

Abstracts are numbered serially throughout this volume.

ALGEBRA AND THEORY OF NUMBERS

453*t*. R. H. Bruck and Erwin Kleinfeld: *The structure of alternative division rings.*

The following theorem is proved: (A) Every alternative division ring R of characteristic other than two is either (i) a field or skew-field or (ii) a Cayley-Dickson division algebra (of order eight over its centre). The essence of the proof is to show that if R, with centre F, is not associative, then every element of R satisfies a quadratic equation over F. This reduces (A) to a theorem of A. A. Albert (*Absolute-valued algebraic algebras*, Bull. Amer. Math. Soc. vol. 55 (1949) pp. 763–768). Call a projective plane alternative if Ruth Moufang's law of the complete quadrangle or Marshall Hall's Theorem L holds universally, but no complete quadrangle has collinear diagonal points. Then (A), together with published work of Moufang, Hall, R. D. Schafer, and others, implies the following theorems: (B) Every alternative plane characterizes a coordinate (alternative division) ring uniquely in the sense of isomorphism. (C) If an ordered plane is alternative it is Desarguesian. That (A) implies (B) was previously pointed out by M. F. Smiley, and (C) answers a question originally posed by Max Dehn to Ruth Moufang some years ago. (This abstract was originally submitted July 27, 1950 to the International Congress, too late for acceptance.) (Received September 15, 1950.)

454*t*. Trevor Evans: *On multiplicative systems defined by generators and relations.* I. *General theory.*

Using the characterization of a loop as a system with three binary operations, a study is made of the properties of loops defined in terms of generators and relations. A normal form theorem is obtained for loops defined by a certain type of relation and it is shown that every finitely related loop can be put in this form. The structure of subloops of loops defined by generators and relations is completely determined and results obtained analogous to those of Grace E. Bates, *The theory of free loops and nets and their generalizations*, Amer. J. Math. vol. 69 (1947) pp. 449–550. Corresponding results hold also for other nonassociative multiplicative systems such as quasigroups, groupoids, and groupoids with unique division on one side. The word problem is solved for loops and the cancellation and division problems for groupoids. (Received August 14, 1950.)

455*t*. Trevor Evans: *On multiplicative systems defined by generators and relations.* II. *Monogenic loops.*

The methods developed in a previous paper by the author *On the theory of multiplicative systems defined by generators and relations.* I. *General theory*, are used to study some properties of monogenic loops (loops generated by one element). The following results are obtained. The group of automorphisms of the free monogenic loop is the free cyclic group. A monogenic loop defined by a finite, nonzero number of relations has only a finite number of endomorphisms. This means that the isomorphism problem can be solved for monogenic loops. Any countable loop can be embedded in a monogenic loop. A monogenic loop is constructed which is isomorphic to a proper factor loop of itself. (Received August 14, 1950.)

456t. D. C. Murdoch: *Intersections of primary ideals in a noncommutative ring.*

Let R be a noncommutative ring in which the ascending chain condition holds for two-sided ideals. An ideal $\mathfrak{q}$ is right primary if when a is not in $\mathfrak{q}$, $aRb\subseteq\mathfrak{q}$ implies $b\in\mathfrak{r}(\mathfrak{q})$, where $\mathfrak{r}(\mathfrak{q})$ is the radical of $\mathfrak{q}$ in the sense of McCoy (Amer. J. Math. vol. 71 (1949) pp. 823–833). The radical of a right primary ideal is prime. If $\mathfrak{a}$ is the intersection of a finite number of right primary ideals then $\mathfrak{a}$ is right primary with radical $\mathfrak{p}$ if and only if all its right primary components have radical $\mathfrak{p}$. Hence any ideal $\mathfrak{a}$ which is the intersection of right primary ideals has a short representation in which the right primary components all have different radicals. In any two short representations of $\mathfrak{a}$ the number of right primary components is the same and the radicals of the components of one representation are equal in some order to the radicals of the components of the other. (Received September 12, 1950.)

457t. H. J. Ryser: *A combinatorial theorem with an application to latin rectangles.*

Let A be a matrix of r rows and n columns composed entirely of zeros and ones, where $1\leqq r<n$. Let there be exactly k ones in each row, and let $N(i)$ denote the number of ones in the ith column of A. If for each i, $k-(n-r)\leqq N(i)\leqq k$, then $n-r$ rows of zeros and ones may be adjoined to A to obtain a square matrix with exactly k ones in each row and column. Let T be an array of r rows and s columns, formed from the integers $1, \cdots, n$ in such a way that the integers in each row and column are distinct. Let $N(i)$ denote the number of times that i occurs in T. The preceding theorem implies that a necessary and sufficient condition in order that T may be extended to an n by n latin square is that for each $i=1, \cdots, n$, $N(i)\geqq r+s-n$. The latter result generalizes M. Hall's existence theorem for latin squares (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 387–388). (Received September 18, 1950.)

ANALYSIS

458t. P. R. Garabedian: *A partial differential equation arising in conformal mapping.*

Let $K(z, t)$ and $k(z, t)$ be the kernel functions in a plane region D which are associated, respectively, with the norms $\iint_D|f(z)|^2\rho dxdy$ and $\iint_D u(x, y)^2\rho dxdy$ of analytic functions $f(z)$ and real harmonic functions $u(x, y)$ in D, where ρ is a positive weight function. These kernels are given in terms of the Green's functions $G(z, t)$ and $g(z, t)$ of the partial differential equations $\partial/\partial\bar{z}(\rho^{-1}\partial/\partial z)G=0$ and $\Delta\rho^{-1}\Delta g=0$ by the formulas $K(z, t)=-2(\pi\rho(z)\rho(t))^{-1}(\partial^2G(z, t)/\partial z\partial\bar{t})$ and $k(z, t)=-(8\pi\rho(z)\rho(t))^{-1}\Delta_z\Delta_t g(z, t)$. The Friedrichs eigenfunctions ψ_ν and eigenvalues μ_ν for $\mu=\left|\iint_D\psi^2dxdy\right|/\iint_D|\psi|^2dxdy=\text{maximum}$, $\partial\psi/\partial\bar{z}=0$, are given in terms of the eigenfunctions U_ν of the partial differential equation $\partial^2U/\partial\bar{z}^2+\mu\partial^2\overline{U}/\partial z\partial\bar{z}=0$ by the formula $(1-\mu_\nu^2)\psi_\nu=\partial U_\nu/\partial\bar{z}+\mu_\nu\partial\overline{U}_\nu/\partial z$. Both problems have the same eigenvalues μ_ν. The eigenfunctions m_ν and eigenvalues λ_ν for the problem $\lambda=\iint_D|\partial m/\partial z|^2dxdy/\iint_D|\partial m/\partial\bar{z}|^2dxdy$