Formeln und Sätze für die Speziellen Funktionen der Mathematischen Physik. By W. Magnus and F. Oberhettinger. (Die Grundlehren der Mathematischen Wissenschaften in Einzeldarstellungen, vol. 52.) 2d. ed Berlin-Göttingen-Heidelberg, Springer, 1948. 8+230 pp. 24.60 D.M.

Much of the toil of mathematical physics consists in the transformation of mathematical expressions which have been arrived at by a conceptual process (of theoretical physics) to other more applicable forms. The book under review is intended to be an aid to this process, and should be of great value to the toilers of mathematical physics.

There are no numerical tables or graphs in the book. Except for the necessary preliminary definitions, the collected material is limited to formulas which are likely to be usable in the solution of special problems. Thus all proofs are omitted. Most physical scientists will probably find the omission of questions relating to expansions in orthogonal systems of functions to be more serious. It does not seem that such an addition would entail much greater length of the text. In that fine synoptic treatment by E. Madelung, *Die Mathematischen Hilfsmittel des Physiker*, one finds the relevant material condensed into twlve pages. A treatment on the same order, but extended to cover the additional functions considered, would make the present book more nearly self-contained, and probably unrivaled in comprehensiveness. A mere repetition of the Madelung material would help. Even old worn descriptions, like Mark Twain's diamonds, are better than none at all.

The following suggestions for future editions may also be made:

(A) Every root of an algebraic equation can be expressed as a hypergeometric function of the coefficients of the equation (a fact considered at least as early as 1914 by Mellin); it would be of value to give the corresponding formulae in the chapter on the hypergeometric function.

(B) There are a few slips in the otherwise excellent typography. The appropriate limits of integration and superscripts are missing in several places, as, for example, on pages 2, 98, and 208.

Of the functions common in mathematical physics, only the Lamé functions are omitted altogether, and the treatment of the Mathieu functions is perhaps unduly brief in view of their growing importance in applications.

Chapter I deals with the gamma function, its logarithmic derivative or Ψ function, and the beta function. Chapter II, on the hypergeometric function, treats of the series and its generalizations, independent solutions of the hypergeometric differential equation, and of its generalized form known as the Riemann differential equation. The specialization of the generalized series on the one hand, and the equation and its solutions on the other, to the many simpler and more familiar forms which they take is concisely and clearly described throughout the book. Chapter III on the cylinder functions, the longest in the book, gives results on the functions of Bessel, Struve, Anger, Weber, Lommel and Mathieu, Kapteyn, Schlömilch, and mixed cylinder-elementary-function types of series are also given.

It is interesting to note that the cylinder functions, for which so much theoretical material has been developed, have also been by far the most worked on numerically. This may be seen by a quantitative survey of the amount of tabular material in a book like Jahnke and Emde's *Funktionentafeln*. In fact, the only outstanding competitor among the non-elementary functions in this matter of popularity with the computers seems to be the family of elliptic and theta functions. If one enlarges the category of cylinder functions to that of the confluent hypergeometric functions, of which it is a special case, one finds that the vast majority of all the numerical-graphical material is shared between the confluent hypergeometric family and the elliptic-theta family. There remains mainly the small number of tables associated with the gamma and Legendre functions.

In Chapter IV on sphere functions (spherical harmonics) the various special forms are discussed, as well as the functions of Gegenbauer in their connection with many-dimensional sphere functions. We remark that an interesting physical interpretation may be given to Hecke's theorem on integrals of functions over hyperspheres (end of Chapter IV). Imagine a general system of dipoles (a *multipole*) at the origin of a unit hypersphere in an n-dimensional space. Suppose further that on the unit hypersphere there is a continuous distribution of charge, arbitrary in every way except that it is rotationally symmetric with respect to a given axis of the sphere. Then the contribution to the total energy, of the continuous charge distribution in the field of the multipole, is entirely equivalent to that for a certain single point charge located at the place where the symmetry axis intersects the hypersphere. Now, the really interesting feature of the theorem is that, no matter how we vary the structure (but not the multiplicity) of the multipole at the center, it is the *same* point charge which may replace a given continuous distribution, as far as its energy contribution is concerned.

Orthogonal polynomials, the confluent hypergeometric function, elliptic integrals, theta functions, and elliptic functions are taken up in the next three chapters. The relations between the various functions are clearly given and transformations and degenerations are carefully listed.

The two last chapters depart from the treatment of special functions to more general considerations. Chapter VIII treats of integral transforms and their inverses. Included are concise accounts and tables of Fourier, Laplace, Hankel, Mellin, and Gauss transforms. All of these transform relations may be regarded as integral equations of the first kind, with essentially singular kernels and infinite domains of integration. Further analogous integral equations are given, among which are several with finite integration interval, for example, Hilbert's cotangent-kernel equation and Abel's integral equation. Chapter IX is a summary of, for the most part, conventional material on coordinate transformations. A novel feature here is the material on the system of many-dimensional polar coordinates.

Several appendices, one on linear second order differential equations, one on Fourier series, partial-fraction and product representations of some elementary functions, and one on certain summations, complete this comprehensive, scholarly, and useful compilation.

M. AVRAMY MELVIN

Sequential analysis. By Abraham Wald. New York, Wiley, 1947. 12+212 pp. $4.00.

This book could be reviewed from the point of view of the general mathematician, the probability theorist, the mathematical statistician, the theoretical statistician, or from the point of view of any of many kinds of users. We shall try to provide a somewhat composite view.

The central feature of this book is the sequential probability ratio test, abstractly a random walk between adsorbing barriers. We have only to think of a "particle" moving in "steps," the amount of each step being determined by chance in the same way, the stepping process ending whenever the particle, which started from the origin, passes certain "barriers" and leaves an assigned interval. Some 40 pages of the mathematical appendix is devoted to exact and approximate results for such walks. The theory of sequential analysis is growing, and more complex processes are entering; yet this simple random walk is still the core.

To read the body of the book, the reader needs, explicitly, a knowledge of calculus, and, implicitly, a feeling of comfort with sentences