The concluding chapter is entitled *Further theory of ideals and commutative rings* and is fairly concentrated. It is concerned with Noetherian rings and algebraic manifolds and is designed, probably, to leave the reader in a more humble frame of mind.

It is surprising that this book and the one by Jacobson (*The theory of rings*, Mathematical Surveys, no. 2, New York, 1943) overlap so little. Doubtless McCoy planned it that way. Jacobson had more space at his disposal and his book has not been supplanted for reference purposes. But the McCoy book has many novel points of view and some more recent material, and as an introduction to the powerful and highly abstract method of thinking which now characterizes modern algebra, it is a gem.

C. C. MacDuffee

Mathematical analysis of binocular vision. By R. K. Luneburg. Princeton University Press, 1947. 6+104 pp. $2.50.

An attempt is made in this study to derive a metric for the psychological space of binocular vision. It is first shown that the recognition of greater and smaller and of greater and smaller contrast uniquely determines the psychometric coordination of numbers to sensation within the limits of a linear transformation. In order to proceed there is then introduced a rather strong hypothesis which is suggested by some experimental observations.

Let an observer first view a point P with head fixed. For this situation a convenient set of coordinates is γ, ϕ, θ, where γ is the angle of convergence (angle LPR, L and R representing the eyes), ϕ is a lateral angular deviation ($PLR/2-PRL/2$), and θ is the ang lar elevation of the plane PRL from the horizontal plane. Next let the observer be permitted to rotate his head about a vertical axis so that the eyes converge symmetrically on P. For this situation another set of coordinates γ^*, ϕ^*, θ^* is introduced. The corresponding angles are very similarly defined. In the transformation from cartesian coordinates to these starred coordinates the distance d' between the line through the eyes and the axis of rotation enters as a parameter.[1]

Now suppose we have two different configurations of object points, the first being viewed with fixed head, the second being viewed with rotating head. If there is a correspondence between the two such that γ for the first equals γ^* for the second and similarly $\phi=\phi^*$, $\theta=\theta^*$,

[1] It would seem that d' could be experimentally made to take on values from zero, or even less, to many times the normal value. If so it would be of interest to determine whether or not the hypothesis would hold when the effect is accentuated by choosing extreme values of d'.

then the hypothesis is that the two configurations lead to the same sensation. Since elements of length appear to be equal, then if the apparent size is determined by a quadratic differential form, it follows that

$$g_{11}d\gamma^2 + g_{22}d\phi^2 + g_{33}d\theta^2 + \cdots = g_{11}d\gamma^{*2} + g_{22}d\phi^{*2} + g_{33}d\theta^{*2} + \cdots,$$

where the coefficients $g_{ik}(\gamma, \phi, \theta)$ on the left are the same functions as $g_{ik}(\gamma^*, \phi^*, \theta^*)$ on the right. This leads to a non-Euclidean metric in general. The observation that two line elements on the same line of view which give the same $d\gamma$, $d\phi$ are seen as parallel, together with considerations of symmetry, leads to the quadratic differential form

$$ds^2 = M^2(\gamma)(\sigma^2 d\gamma^2 + d\phi^2 + \cos^2 \phi d\theta^2),$$

in which σ is a parameter of the observer which measures the relative effectiveness of convergence as compared with angular displacement in the estimation of length. Further considerations suggest that $M(\gamma) = 1/\sin h\sigma(\gamma + \mu)$, in which case the metric is hyperbolic.

Some of the topics which are treated in considerable detail are the horopter problem (geodesic lines), the alley problem, and rigid transformations of the hyperbolic visual space. In the alley problem with walls sensed as parallel it is pointed out that the meaning of "parallel" is ambiguous. Two cases are treated which correspond to different instructions to the subject. These in general give different results, the difference depending on the geometry chosen. Thus, while it is possible to account for apparently conflicting experimental results, it is also possible to make use of the experimental data to obtain restrictions on the metric. Using the hyperbolic metric, the author calculates the shape of distorted rooms which are congruent to rectangular rooms, that is, rooms with distorted walls and windows which appear (under fixed conditions) to be identical to rectangular rooms with rectangular windows.

Even if further experiments show that the metric derived is not adequate to account for the data, the author's efforts will greatly facilitate the task of determining a better approximation. The point of view developed, together with the many suggestions given, should prove to be of great help in determining the direction to be taken for further theoretical studies and experimental observations.

H. D. LANDAHL

Leçons de géometrie différentielle. Vol. 1. By G. Vranceanu. Bucarest, Rotativa, 1947. 422 pp.

This volume, which is the first of two proposed volumes, is divided