

## BOOK REVIEWS

*Advanced calculus.* By D. V. Widder. New York, Prentice-Hall, 1947. 16+432 pp. \$5.00.

This text gives careful treatments of a restricted selection of topics culminating in the theory of real Laplace transforms.

*Content:* Limits and derivatives of functions of one variable are sketched in four pages. The treatment of partial derivatives emphasizes the theory of functional relations. The differential geometry of curves and surfaces is developed after a short chapter on vectors. A thorough treatment is given to maxima and minima. Single integrals are developed in the form of Stieltjes integrals, with Riemann integrals merely mentioned as a special case. But multiple integrals are given a relatively conventional treatment including some of the usual elementary applications. The theory of line and surface integrals is carried as far as the basic integral relations. Indeterminate forms are handled rather conventionally. The chapters on infinite series and improper integrals are closely parallel. They begin with a relatively detailed treatment of the elementary convergence tests; then follow the theory and applications of uniform convergence; the discussion closes with an introduction to the process of summation. Brief treatments are given to the gamma and beta functions, and to Stirling's formula. Various topics are included in the theory of Fourier series and integrals. The last two chapters develop the theory of real Laplace transforms and their applications to the solution of differential and difference equations. Such subjects as complex variables, matrices, variations, numerical methods, and statistics are omitted entirely; differential equations are only touched on in connection with line integrals and Laplace transforms.

*Style:* The formulation of most of the definitions and theorems is illustrated by the first numbered definition in the book:

DEFINITION 1.  $f(x) \in C$  at  $x = a \leftrightarrow \lim_{x \rightarrow a} f(x) = f(a)$ .

Precision and rigor constitute an outstanding feature of this text. The extent to which these attributes are carried is illustrated by the following definition (page 30):

DEFINITION 8. *The directional derivative of  $f(x, y)$  in the direction  $\xi_\alpha$  at  $(a, b)$  is*

$$\left. \frac{\partial f}{\partial \xi_\alpha} \right|_{(a,b)} = \lim_{\Delta s \rightarrow 0} \frac{f(a + \Delta s \cos \alpha, b + \Delta s \sin \alpha) - f(a, b)}{\Delta s}.$$

Great care is taken to show the relation between hypothesis and conclusion. Effective use is made of "counter examples." There are many exercises.

The amount of space devoted to specific applications is indicated by the following numbers of pages: mercator maps, 1; least squares, 1; mass, center of gravity, moment of inertia, and force of attraction, 7; work, 1; vibrating strings, 8.

*Criticisms:* Only a few misprints were noted, such as: p. 13, Case II,  $s = \varphi(r, s, t)$  should read  $x = \varphi(r, s, t)$ ; p. 101, Theorem 1,  $C'$  should be  $C^1$ ; p. 263, line 1,  $S_n$  should be  $\sigma_n$ . On page 5, and in the Index of Symbols, "not" is denoted by  $\mid$ , whereas in the text, "not" is denoted by  $/$ .

The text contains only 40 figures, many of which are rudimentary.

Italic letters are used for both scalar and vector variables. (While this is logically sound, it is a questionable psychological hazard.)

The author's English tends at times to be cryptically terse. For example, Exercise 8 on page 328 reads: Prove the rest of the orthogonality and normality relations.

On page 50 the author states: By a *vector* we mean a directed line segment. Farther down this page the author writes: DEFINITION 1. A *vector*  $r$  is a triple of numbers  $(r_1, r_2, r_3)$ . A similar difficulty occurs in connection with homogeneity (p. 14) and  $\nabla$  (p. 65).

*Conclusion:* Students who can adapt themselves to Professor Widder's style will surely find this text to be elegant and cogent, and an admirable introduction to the finesse of mathematical methods.

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*Antennae: an introduction to their theory.* By J. Aharoni. New York, Oxford University Press, 1947. 8+265 pp. \$8.50.

Antenna theory is a promising field of activity for mathematicians of varying degrees of expertness and "purity." Most of its boundary value problems and all of its integral equations are both difficult and intriguing. Antenna theory offers an opportunity for fundamental contributions both to the theory of partial differential equations and to the theory of integral equations of the first kind. Unfortunately it is not easy for the mathematician to become acquainted with antenna problems, for most books on the subject he would find unreadable. The present volume by J. Aharoni may remedy this situation. Although *Antennae: an introduction to their theory* is not written primarily for mathematicians as such or for the purpose of stimulating their interest in antenna problems, it is a book which is likely to be intelligible to them. Neither special engineering background nor