BOOK REVIEW

Vorlesungen über Differential- und Integralrechnung. Vol. 1. *Funktionen einer Variablen.* By A. Ostrowski. Basel, Birkhäuser, 1945. 12+373 pp. 47.50 s.fr.

According to the author's statement, this volume is intended to serve as the basis of a one semester course introducing the student to the fundamental concepts and techniques of analysis. The book consists of seven chapters. Chapter I is devoted to a treatment of the real number system, not as something built up in stages from the integers, but as a system described axiomatically. Mathematical induction, ordered fields, and Dedekind cuts are discussed carefully and at length. In Chapter II the limit concept for sequences and functions is introduced, first for convergence to zero, and then in general. Chapters III and IV introduce the two fundamental concepts of calculus. The definition of the definite integral precedes that of the derivative. A proof, employing Dedekind cuts, is given of the existence of the integral of a continuous function, but the author suggests that the student may very well pass over the proof lightly at first. The development of technique in differentiation and integration is attended to in Chapters V and VI. These chapters also contain discussions of monotone functions and their inverses, and of logarithmic and exponential functions. Curiously, the general development in Chapter III is interrupted by placing a section on the trigonometric functions between the treatment of continuous functions and the definition of the definite integral. The book concludes with a chapter containing the following topics: uses of the first and second derivatives in studying the properties of functions and in tracing curves; indeterminate forms; series expansions of the logarithm and the inverse tangent; Taylor's formula with remainder; Euler's formulas.

The emphasis throughout is upon hypotheses, definitions, and fundamental concepts. The book begins with a brief discussion of the characteristics of mathematics as a discipline. There are some good observations about the importance of clear definitions and about the necessity for proof even of the intuitively obvious. The proofs of certain theorems (the properties of functions continuous on a closed interval, and the sufficiency of the Cauchy convergence criterion) are omitted. It is interesting to note, in comparison with American practice in introductions to calculus, that nearly one fourth of the book is taken up with the subject matter of the first two chapters alone.

Roughly one fourth of the book is devoted to exercises. Virtually all of these pertain strictly to analysis; there are no applications to physics or the other sciences, nor to the calculation of volumes and arc lengths. Analytic geometry is used sparingly. There are relatively few (48) figures; these are uneven in quality.

Several interesting and unusual features deserve brief mention. There is a short discussion of the possibility of defining the inverse trigonometric functions by definite integrals, and then studying the trigonometric functions as the inverses of the functions so defined. The history of elliptic integrals and the theory of elliptic functions is cited in this connection (pp. 206–207). There is a beautifully simple discussion of the law of natural growth (pp. 262–263). There is a more than usually full discussion of the computing of logarithms by series (pp. 337–340).

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