then the sequence $f(nt)$ is complete in $L_\infty(0, \pi)$. (3) Let $f(t) = t - [t] - 1/2, f(n) = 0$; the sequences $\{f(nt)\}$ and $\{\sin 2n\pi t\}$ have the same span in $C(0, 1/2)$. (4) $f(t) = \operatorname{sgn} \sin n\pi t$; the sequence $\{f(nt)\}$ is complete in $L_r(0, 1)$ for all $r > 1$. (5) Each of the sequences $1, e^{-t}, t/(e^{nt}-1)$; $1, e^{-t}, t^2/(e^{nt}-1)^2$, $n = 1, 2, 3, \cdots$, is complete in $C(0, \infty)$. (Received May 31, 1946.)

241. Hing Tong: *Ideals of normed rings associated with topological spaces*. Preliminary report.

Let R be a perfectly normal bicompact space, $\mathfrak{R}$ the normed abelian ring of complex-valued functions continuous over R (for $f \in \mathfrak{R}$, $\|f\| = \max_R |f(x)|$), $\mathfrak{J}$ any (closed) ideal in $\mathfrak{R}$. Then $\mathfrak{J}$ is a principal ideal. If $\prod_{f \in \mathfrak{J}} \mathfrak{N}_f = N_{\mathfrak{J}}$ ($\mathfrak{N}_f$ denotes the set of zeros of f) corresponds to $\mathfrak{J}$, the correspondence is an isomorphism (that is, $\mathfrak{J} \leftrightarrow N_{\mathfrak{J}}$) such that ($\alpha_1$) $\mathfrak{J}_1 \cup \mathfrak{J}_2 \leftrightarrow N_{\mathfrak{J}_1} \cdot N_{\mathfrak{J}_2}$, ($\alpha_2$) $\mathfrak{J}_1 \cap \mathfrak{J}_2 \leftrightarrow N_{\mathfrak{J}_1} + N_{\mathfrak{J}_2}$, ($\alpha_3$) $\mathfrak{R} \leftrightarrow \{0\}$, ($\alpha_4$) $\sum_\alpha \mathfrak{J}_\alpha \leftrightarrow \prod_\alpha N_{\mathfrak{J}_\alpha}$, ($\alpha_5$) $\prod_\alpha \mathfrak{J}_\alpha \leftrightarrow$ closure of $\sum_\alpha N_{\mathfrak{J}_\alpha}$. Conversely, let $\mathfrak{R}$ be the normed ring of bounded complex-valued functions continuous over a topological space R. The conditions $\mathfrak{J} \leftrightarrow N_{\mathfrak{J}}$, ($\alpha_1$) holds, and every $\mathfrak{J} \subseteq \mathfrak{R}$ is principal imply that R is a perfectly normal bicompact space. If R is a T_1' (a space $\ni y \in \bar{x} \rightarrow \bar{x} = \bar{y}$) bicompact normal space, the above results hold providing "principal ideal" is replaced by "ideal." The results also hold for bicompact normal spaces if isomorphism is relaxed to homomorphism. The ring of continuous mappings (continuous in the strong topology) of R into the ring of bounded linear operators over a Banach space with a basis leads to the same results if an ideal means a two-sided ideal. The results do not hold for non-separable Banach spaces. (Received April 10, 1946.)

Applied Mathematics

242. Garrett Birkhoff: *Symmetric Lagrangian systems*.

Let Ω be any Lagrangian dynamical system with no potential energy and kinetic energy function $L = 2^{-1} E_{jk} \dot{q}_j \dot{q}_k$. Suppose that L is invariant under a simply transitive group G of rigid motions on its configuration space. Then the "generalized force" required to maintain motion along a one-parameter subgroup in the jth coordinate direction has the components $Q_i = c^{ij}_{\ k} E_{jk}$, where the $c^{ij}_{\ k}$ are the structure constants of G. It is a corollary that the d'Alembert paradox would take in non-Euclidean geometry the following form. A rigid body moving under translation, rotation, or screw motion along an axis in an incompressible, nonviscous fluid without circulation will experience no thrust along or torque about the axis. However cross-force is possible. (Received May 13, 1946.)

243. R. J. Duffin: *Nonlinear networks*. III.

A system of n nonlinear differential equations is shown to have a periodic solution. The interest of these equations is that they describe the vibrations of electrical networks under a periodic impressed force. Consider an arbitrary linear network of inductors, resistors and capacitors which does have a solution for a given periodic impressed force. The main result of this note states that the existence of a periodic solution is still guaranteed if the linear resistors of such a network are replaced by *quasi-linear* resistors. A quasi-linear resistor is one whose differential resistance lies between positive limits. No other sort of nonlinearity besides this type of nonlinear damping is considered. The proof rests on the closure properties of nonlinear transformations in Hilbert space. (Received May 7, 1946.)

244. G. F. McEwen: *Diffusion from an instantaneous circular area source.* Preliminary report.

Corresponding to an initial concentration S_1 within a circular area of radius r_1 and constant coefficient of diffusion a^2, the partial differential equation $\partial S/\partial t = a^2(\partial^2 S/\partial r^2 + r^{-1}\,\partial S/\partial r)$ has the formal solution $S = S_1 r_1 \int_0^\infty J_1(\lambda r_1) J_0(\lambda r) \exp(-a^2 t\lambda) d\lambda$, which reduces to $S = S_1[1 - \exp(-r_1^2/4a^2t)]$ for $r=0$. Differentiation of the formal solution with respect to r yields an expression whose integral with respect to λ, from 0 to ∞, is known. The general solution in a form adapted to computation is found by integrating this expression from 0 to r, $\int_0^r I_1(rr_1/2a^2t)\exp(-r^2/4a^2t)dr$, and adding the solution for $r=0$. Repeated integration by parts in two ways reduces this integral to corresponding converging series and leads to the solutions: $S = S_1\{1 - \exp[-(r-r_1)^2/4a^2t]\sum_{n=0}^\infty (r/r_1)^n f_n(rr_1/2a^2t)\}$ for $r \leqq r_1$, and $S = S_1 \exp[-(r-r_1)^2/4a^2t] \cdot \sum_{n=1}^\infty (r_1/r)^n f_n(rr_1/2a^2t)$ for $r \geqq r_1$, where $f_n(z) = e^{-z} I_n(z) \leqq 1$. For a diffusion coefficient $a^2 r$ the equation $\partial S/\partial t = a^2(r\partial^2 S/\partial r^2 + 2\partial S/\partial r)$ has, for the same boundary conditions, the formal solution, $S = 2S_1 r_1/r^{1/2} \int_0^\infty J_2(2(r_1)^{1/2}\lambda) J_1(2r^{1/2}\lambda)\exp(-a^2 t\lambda) d\lambda$, which yields two solutions: $S = S_1\{1 - (r_1/r)^{1/2}\exp[-(r^{1/2} - (r_1)^{1/2})^2/a^2t] f_1(2(rr_1)^{1/2}/a^2t) - \exp[-(r^{1/2} - (r_1)^{1/2})^2/a^2t]\sum_{n=0}^\infty ((r/r_1)^{1/2})^n f_n(2(rr_1)^{1/2}/a^2t)\}$, and $S = S_1 \exp[-(r^{1/2} - (r_1)^{1/2})^2/a^2t]\sum_{n=2}^\infty ((r_1/r)^{1/2})^n f_n(2(rr_1)^{1/2}/a^2t)$. (Received May 10, 1940.)

245. H. E. Salzer: *Alternative formulas for direct interpolation of a complex function tabulated along equidistant circular arcs.*

An improved method of complex Lagrangian interpolation for an analytic function that is tabulated along the arc of a circle at equidistant intervals is obtained by generalization of a scheme recently described by W. J. Taylor. In place of Taylor's binomial coefficients, for n-point direct interpolation, auxiliary quantities $A_k^{(n)}$, where $n = 3, 4, \cdots, 11$ and $k = -[(n-1)/2]$ to $[n/2]$, are given as functions of the angular interval θ, so that for extensive use for some fixed θ, they can be found as fixed complex numbers. In this method of complex interpolation the number of operations increases linearly with n, whereas in the author's previous method (which employed Lagrangian interpolation polynomials explicitly and was cumbersome even for the 5-point case) the number of operations increased quadratically with n. Advantage is taken of the fact that, for each n, all the quantities $A_k^{(n)}$ can be multiplied by any nonzero factor in order to simplify their expression. It is proved that for odd n, $A_{-k}^{(n)}$ equals conjugate of $A_k^{(n)}$, and for even n, $A_{k+1}^{(n)} = -\exp(i\theta\{k(2-n) - n/2 + 1\})A_{-k}^{n}$. (Received May 14, 1946.)

246. S. A. Schaaf: *On the superposition of a heat source and a contact resistance.*

The Laplace transform is used to determine the temperature distributions $T_i(x, t)$ $(i = 1, 2)$ in two semi-infinite heat conductors, initially at zero temperature and in contact along the interface plane $x=0$ where there is superposed a heat source of strength $S(t)$ and a contact resistance. It is shown that such a superposition leads in many cases (including those in which the heat source is caused by friction as the two solids slide against each other along the interface) to a boundary condition at $x=0$ of the type $T_1 - T_2 = R(C_1 S + K_1 \partial T_1/\partial x) = -R(C_2 S + K_2 \partial T_2/\partial x)$, where R is the resistance constant, K_i the heat conductivity and C_i a constant whose interpretation depends on the physical model of the interface. (Received May 16, 1946.)

247. A. C. Sugar: *An elementary exposition of the relaxation method.*

This is, as labeled, an elementary exposition. The purpose of this paper is to exhibit the simplicity and power of the relaxation method, to complete and explain previous sketchy or obscure discussions of this subject. It is also intended to direct attention to some mathematical, physical, and philosophical questions that may be raised in connection with this method. (Received May 29, 1946.)

248. A. C. Sugar: *On the relaxation-matrix method of solving boundary value problems.*

This is a continuation of the study of the use of relaxation and iterative methods of inverting the matrices of the systems of equations obtained from boundary value problems by finite difference methods. In a previous paper, entitled *The use of invariant inverted matrices for the approximate solution of classes of boundary value problems,* the writer inverted matrices by relaxation methods and applied them to the solution of simple illustrative problems. In the present paper, this work is continued and applications are made to some of the typical boundary value problems of mechanics. (Received May 29, 1946.)

249. A. C. Sugar: *The use of invariant inverted matrices for the simultaneous approximate solution of classes of boundary value problems.* Preliminary report.

The writer considers the simultaneous approximate solution of classes of boundary value problems. Each class consists of the totality of boundary value problems having the same differential equation and the same boundary but different boundary conditions. Using finite difference methods it is shown that the derived system of equations will have an inverse matrix M, invariant over the class, which may be determined by relaxation methods. A solution of any member of the class may then be obtained by multiplying M by a column matrix defined by the corresponding boundary values. This paper will be primarily concerned with the application of this method to Laplace's equation. The effect of modifications of the boundary on M will be considered. This technique may be applied to many other types of differential equations. This is true, in particular, of Poisson's equation and of nonlinear equations containing the Laplace operator, since, as far as the algebraic treatment is concerned, these two types may be treated as Laplace equations with altered boundary conditions. Finally, the possibility of considering M as an approximation or an analogue of Green's function is studied. (Received April 16, 1946.)

GEOMETRY

250. L. M. Blumenthal: *Superposability in elliptic spaces.*

Two subsets of a metric space M are *superposable* provided a congruent (that is, one-to-one, distance-preserving) mapping of M onto itself exists which maps one subset onto the other. In spaces most studied (euclidean, spherical, and so on) congruence of subsets implies that they are superposable, but this is not the case in elliptic spaces $E_{n,r}$ for $n>1$, and hence this property cannot be expressed in metric terms alone. This paper shows that congruent but not superposable subsets of $E_{n,r}$ fall into two classes (a) congruent subsets contained irreducibly in different dimensional subspaces and (b) those contained irreducibly in subspaces of the same dimension. By means of