tives of a general polygenic function. This leads to the geometry of the related circles, limaçons, and cardioids. (When the function is monogenic, these related curves degenerate into points.) In addition to summarizing the already published material, many new theorems are included. (Received October 26, 1944.)

39. V. G. Grove: *Quadrics associated with a curve on a surface.*

The quadrics of Darboux, Moutard and Davis, the conjugal quadrics, the asymptotic osculating quadrics and many other quadrics belong to a certain family of quadrics. This paper seeks to characterize all of the members of this family in terms of cross-ratios. In so doing a generalization is obtained for Bell's R-associate of a line in the tangent plane. Some special quadrics of the pencil are characterized and new characterizations of the pan-geodesics are obtained. (Received October 7, 1944.)

40. C. C. Hsiung: *A ternary of plane curvilinear elements with a common singular point.*

This paper studies three curves having a common singular point of different kinds and a common tangent at the point. A projective invariant is found and characterizations are found for the invariant for various kinds of singularities. (Received October 7, 1944.)

41. Edward Kasner: *Multi-valued symmetries.*

The author studies conformal symmetry in a general algebraic curve. This is equivalent to Schwarzian reflection for an analytic curve. For an algebraic curve of degree n, the operation T is in general of degree n^2. The degrees of the powers of T are studied in detail. In the special case of a conic, the results are noteworthy. If the base curve is a potential curve (obeys the Laplace equation), symmetry is easily constructible. Satellite curves discussed in a previous paper are related to the present theory. (Received October 26, 1944.)

42. E. J. Purcell: *Some Cremona involutions in n-dimensional space.*

A previous paper (E. J. Purcell, *Variety congruences of order one in n-dimensional space*, Amer. J. Math. vol. 66 (1944) pp. 621–635) discusses linear k-parameter systems of varieties in n-dimensional projective space (k any positive integer not greater than n). Each variety of such a system is of dimension $n-k$ and order h (h any positive integer). Through a generic point of $[n]$ one and only one variety of the system passes. When $n=k$ and $h=2$, a generic variety of the system is a pair of points. Each point determines the pair to which it belongs and the system consists of the pairs of a rational Cremona involution in $[n]$. This paper treats a type $(n)_n$ Cremona involution in $[n]$. When $n=2$, the involution is Geiser's. When $n=3$, the involution is due to Sharpe and Snyder. (Received October 25, 1944.)

Statistics and Probability

43. T. R. Hollcroft: *The probability of repetitions.*

The probability of repetitions is concerned with repetitions only and not with the particular numbers that are repeated. For example, let one number be drawn at a time from ten and replaced after each draw. Eight may be drawn as follows: 4 3 7 6 4 7 6 7. This set contains one triple and two double repetitions. The double repetition 4 4 counts merely as a pair as does 6 6 or any other two of a kind drawn. This set is represented by the symbol (3221). The repetitions and number of trials in the set characterize the set. Let n be the number of trials and p be the probability of drawing a given number in one trial. The total number of sets of repetitions for any n are given in a table. The total number of ways in which each set can be drawn is then found and finally general formulas for the probability that a given set will occur in n trials. Relations among probabilities are obtained. A special case of one of these states that two pairs are $3(n-3)/4$ times as probable as one triple. Similar probability formulas are derived in case the draws are made without replacement. (Received October 10, 1944.)

44. Mark Kac: *Random walk in the presence of absorbing barriers.*

The solution of the random walk problem in the presence of absorbing barriers depends on calculating probabilities of the form $P(n;\ p,\ q) = \text{Prob.}\{-q \leqq X_1 \leqq p,\ -q \leqq X_1 + X_2 \leqq p, \cdots, -q \leqq X_1 \cdots X_n \leqq p\}$, where $X_1, X_2, \cdots$ are identically distributed random variables and $p \geqq 0$, $q \geqq 0$. The present paper illustrates a method by means of which explicit formulas can be obtained. If each X assumes values 1 and -1 with probability 1/2 and if p and q are integers one obtains $P(n;\ p,\ q) = 2(p+q+2)^{-1}\sum \cos^n \pi j/p+q+2 \sin \pi(q+1)j/p+q+2 \cot \pi j/2(p+q+2)$, where j runs from 1 to $p+q+1$ through odd integers only. If the density of the probability distribution of each x is $\exp(-|x|)/2$, $P(n;\ p,\ q)$ can be expressed in terms of the roots of the transcendental equation $\tan(p+q)y = -2y/1-y^2$. The method of solution depends on the use of the eigenvalue theory of matrices (discrete case) and integral equations (continuous case). (Received November 21, 1944.)

45. A. T. Lonseth: *A note on relative errors in systems of linear equations.*

A nonsingular system of linear equations is considered, the coefficients being subject to error. Under the assumption of uniformly bounded errors in the coefficients, a bound is found for the relative error in each solution-component. (This note is to appear soon in the Annals of Mathematical Statistics.) (Received October 19, 1944.)

46. A. T. Lonseth: *Error-limitation for the method of least squares.*

Let $y(t)$ be such that $\int_0^1 y^2 dt = \|y\|^2$ exists. Let $x(t)$ satisfy $Tx = y$, where T is an additive, homogeneous bi-unique transformation possessing the bounded inverse T^{-1} with bound $M(T^{-1})$: that is, $\|T^{-1}y\| \leqq M(T^{-1})\|y\|$ for every y with finite $\|y\|$. Omitting boundary conditions, which enter if T is a differential operator, an estimate $x_n^0(t)$ is obtained as follows (method of least squares): given a set of n functions $\phi_k(t)$ such that $\|T\phi_k\|$ exists for $k = 1, 2, \cdots, n$, values $a_1^0, a_2^0, \cdots, a_n^0$ of parameters $a_1, a_2, \cdots, a_n$ are determined so as to minimize $F(a_1, a_2, \cdots, a_n) = \int_0^1 (Tx_n - y)^2 dt$, where $x_n = \sum_{i=1}^{i=n} a_i\phi_i$. Then $x_n^0 = \sum_{i=1}^{i=n} a_i^0 \phi_i$. It is shown in this paper that the *global error* $\|x_n^0 - x\| \leqq M(T^{-1})F(a_1^0, a_2^0, \cdots, a_n^0)$, where $F^2(a_1^0, a_2^0, \cdots, a_n^0) = \|y\|^2 - \sum_{i=1}^{i=n} a_i^0 \int_0^1 yT\phi_i dt$. (The integrals under summation occur in the linear equations for the minimizing parameters.) For linear integral equations of Fredholm type and second kind the *local error* $|x_n^0(t) - x(t)|$ is similarly bounded. If T is continuous, hence itself bounded, the *accuracy* of the approximation can be limited: $\|x_n^0 - x\| \geqq M(T)^{-1}F(a_1^0, a_2^0, \cdots, a_n^0)$. (Received October 19, 1944.)

47. Henry Scheffé: *A note on the Behrens-Fisher problem.*

Let $(x_1, \cdots, x_m)$ and $(y_1, \cdots, y_n)$ be samples from two normal universes. The problem is the comparison of the means of the universes when the ratio of their variances is unknown. Solutions based on the t-distribution were studied in a previous paper (Annals of Mathematical Statistics vol. 14 (1943) pp. 35–44), and a very convenient one was singled out. This, however, did not have the desirable "symmetry" property, that is, invariance under permutations of the x's among themselves and of the y's among themselves. This note outlines a proof that there exists no "symmetric" solution based on the t-distribution. (Received October 26, 1944.)

Topology

48. L. M. Blumenthal: *Generalized elliptic spaces and quadratic forms.*

Continuing his study of generalized elliptic spaces (abstract 49-11-304) the writer obtains quadratic form theorems concerning invariance of rank under sign changes of coefficients. Thus, for example, if a quadratic form in more than three variables, with the coefficients of the squared terms 1 and those of the products x_ix_j $(i<j)$ between -1 and 1, is positive definite of rank 2, then each positive definite form obtained by a change of sign of coefficients of terms x_ix_j is also of rank 2. The "natural" extension of this theorem to forms in more than four variables with rank 3 is not valid. By investigating the different kinds of equilateral sets contained in n-dimensional elliptic space, the writer shows that these spaces have neither congruence order $n+3$ nor $n+4$ (except for $n=1$). Since, for example, the ordinary elliptic plane contains an equilateral 6-point it failed to have congruence order 6. (Received October 23, 1944.)

49. R. H. Fox: *An application of the complete homotopy group.*

The fundamental group $\tau_n(Y)=\pi_1(Y^T)$ of the space of continuous mappings of the $(n-1)$-dimensional torus $T=T_{n-1}$ into a topological space Y was introduced in a previous communication (abstract 49-11-306). J. H. C. Whitehead has proved two theorems (Proc. London Math. Soc. (2) vol. 45 (1939) p. 281 and Ann. of Math. vol. 42 (1941) p. 418) about homotopy groups. When these are combined and restated in terms of the groups τ_n the result is as follows: Let K^* be a complex and let K be a complex obtained from K^* by removing the interior $\sigma-\dot{\sigma}$ of a principal n-dimensional simplex σ, where $n>2$. The nucleus of the injection homomorphism $\tau_n(K)\rightarrow\tau_n(K^*)$ is precisely the invariant subgroup of $\tau_n(K)$ which is generated by the image of the injection homomorphism $\tau_n(\dot{\sigma})\rightarrow\tau_n(K)$. This reformulation seems to have more intuitive content and suggests an attack on the harder problem where dim $\sigma>n$. (Received October 28, 1944.)

50. R. H. Fox: *On topologies for function spaces.*

To be useful a topology for the set F of continuous functions from X to Y should have one or more of the following properties: (1) A function from $X\times T$ to Y is continuous if and only if the corresponding function from T to F is continuous. (2) The sectioning operation to or from F is continuous. (3) The function $\phi(x, f)=f(x)$ from $X\times F$ to Y is continuous. For any compact set $A\subset X$ and open set $W\subset Y$ let $M(A, W)$ denote the set of continuous functions $f\in F$ such that $f(A)\subset W$. The topology determined by the subbasis $\{M(A, W)\}$ satisfies (1) and (2) if $X\times T$ is separable Hausdorff. If X is normal but not locally compact and Y is an arc, no topology satisfies both (1) and (3). (Received October 21, 1944.)