tion of the random variables x and y where x corresponds to the values assumed by the unknown under test and y corresponds to the values assumed by the control from test to test. (Received March 23, 1944.)

163. P. R. Halmos: *Randon alms.*

Suppose that a pound of gold dust is distributed at random among a countably infinite set of beggars, so that the first beggar gets a random portion of the gold, the second beggar gets a random portion of the remainder, and so on ad infinitum. The purpose of this work is to calculate the actual and the asymptotic distributions of x_n and S_n (where x_n is the amount received by the nth beggar and $S_n = \sum_{i \leqq n} x_i$) and also to study the rate of convergence of the random series $x_1 + x_2 + x_3 + \cdots$, under the assumption that the phrase "random portion," occurring an infinite number of times in the description of the stochastic process, receives the same interpretation each time. The results may be interpreted as properties of a random distribution of a unit mass on the positive integers; they may be used to explain the experimentally observed distribution of sizes of mineral grain particles; and they occur also as distributions of energy in the theory of scattering of neutrons by protons of the same mass. (Received March 24, 1944.)

164. Henry Scheffé and J. W. Tukey: *Contributions to the theory of non-parametric estimation.*

For problems of non-parametric estimation, concerning an unknown cumulative distribution function $F(x)$, three good solutions are available: (i) confidence intervals for the median of F (W. R. Thompson, K. R. Nair), (ii) tolerance limits for F (Wilks), and (iii) confidence limits for F (Wald, Wolfowitz, Kolmogoroff). Heretofore (i) and (ii) have been limited to the case where $F'(x)$ is known to be continuous. By means of a theorem of general application they are extended to the case where F need only be continuous. The only previous result for discontinuous F is that of Kolmogoroff for (iii). The appropriate modifications of (i) and (ii) extending their validity to this case are found. Some uniqueness results limiting the kind of statistics usable in such problems are obtained. Sufficiently complete tables for applying (i) and (iii) have been published, but computations for (ii) have been extremely few and laborious. A simple formula based on the z and χ^2-distributions is found which gives highly accurate approximations in ranges of practical interest. All the results for (ii) also apply to Wald's tolerance intervals for the multivariate case. (Received March 14, 1944.)

Topology

165. E. F. Beckenbach and R. H. Bing: *On generalized convex functions.*

Let $F(x; \alpha, \beta)$ be a two-parameter family of real continuous functions defined in an interval (a, b) such that there is a unique member of the family taking on arbitrary values y_1, y_2 at arbitrary distinct points x_1, x_2 of the interval. A real function $f(x)$ is said to be a sub-$F(x; \alpha, \beta)$ function in (a, b) provided at the midpoint x_0 of each subinterval I of (a, b) we have $f(x_0) \leqq F(x_0; \alpha, \beta)$ for that $F(x; \alpha, \beta)$ which coincides with $f(x)$ at the endpoints of I. The family $F(x; \alpha, \beta)$ is not necessarily topologically equivalent to the set of non-vertical line segments in the strip; hence the study of sub-$F(x; \alpha, \beta)$ functions is not topologically equivalent to the study of convex functions. It is shown among other things that if a sub-$F(x; \alpha, \beta)$ function $f(x)$ is bounded, or even measurable, in a subinterval of (a, b), then $f(x)$ is continuous. (Received March 31, 1944.)

166. Samuel Eilenberg: *Skew-invariant cohomology groups.* Preliminary report.

Let X be a topological space with a group W acting as a group of homeomorphisms on X, and let G be an abelian group with W as a group of operators. An n-dimensional cochain f on X with coefficients in G is called skew-invariant provided $f(wT)=wf(T)$ for every singular n-simplex T in X and every $w\in W$. If f is skew-invariant, then so is its coboundary δf. Using *skew-invariant* cochains throughout, the skew-invariant cohomology group $I_n(X, G)$ is defined. Let Y be an arcwise connected space, locally connected in dimension 0 and 1, and let the fundamental group $\pi_1(Y)$ of Y act as a group of operators on the coefficient group G. Let $H_n(Y, G)$ denote the nth cohomology group of Y with G as a group of local coefficients as recently defined by Steenrod (Ann. of Math. vol. 44 (1943) pp. 610–627). Let $\tilde{Y}$ be the universal covering of Y. The group $\pi_1(Y)$ acts on $\tilde{Y}$ as a group of homeomorphisms, namely, the covering homeomorphisms, and one may consider the skew-invariant group $I_n(\tilde{Y}, G)$. The main theorem asserts that $H_n(Y, G)\approx I_n(\tilde{Y}, G)$. (Received March 3, 1944.)

167. R. C. James: *Linear functionals and orthogonality in normed linear spaces.*

Of several definitions of orthogonality in normed linear spaces, perhaps the most interesting is: "$x\perp y$ if and only if $\|x+ky\|\geqq\|x\|$ for all k." For any elements x and y there exist numbers a and b such that $x\perp(ax+y)$ and $(bx+y)\perp x$. Since this orthogonality is not symmetric, it is not necessary that $a=b$. The uniqueness of this number a for all x and y $(x\neq 0)$ is equivalent to the Gateaux differentiability of the norm at each nonzero point. The uniqueness of b is equivalent to strict normability. If T is a uniformly convex Banach space and L a linear subset not dense in T, there exists an element x orthogonal to L. From this it follows that for every linear functional on T with modulus 1 there exists an element x such that $|f(x)|=\|x\|$. If the norm of T is Gateaux differentiable at all nonzero points, then every linear functional is a Gateaux differential of $\|x\|$ at some point x_0. (Received March 31, 1944.)

168. N. E. Steenrod: *The classification of sphere bundles.*

The problem of Whitney of classifying k-sphere bundles over a complex B as base space is reduced to a familiar problem of topology as follows. Let S be a $(k+l+1)$-sphere ($l=\dim B$), R its rotation group, S' a fixed k-sphere in S, R' the subgroup of R mapping S' on itself, and M_l^k the space of left cosets of R' in R. To each map g of B in M_l^k is attached the k-sphere bundle $A(g)$ over B which is the subset $\{(b, s)\}$ of $B\times S$ such that s lies in the image of S' under any rotation of $g(b)$. The function $A(g)$ induces a 1-1 correspondence between the equivalence classes of k-sphere bundles over B and the homotopy classes of maps of B in M_l^k. The homotopy groups of M_l^k are computed for dimensions 1 through 6 for all k, l. This leads to a complete solution of the classification problem for B a sphere of dimension less than or equal to 6. (Received April 1, 1944.)

169. G. S. Young: *A generalization of Moore's theorem on simple triods.*

If n is a non-negative integer, by a T_n-set is meant a continuum which is the sum of an n-cell, c, and an arc, a, such that $c \cdot a$ is a point which is an end point of a and an interior point of c. A T_1-set is a simple triod. In this note it is proved that Euclidean n-space does not contain uncountably many mutually exclusive T_{n-1}-sets. For $n=2$, this is a theorem due to Moore (Proc. Nat. Acad. Sci. U.S.A. vol. 14 (1928) pp. 85–88). (Received March 27, 1944.)

170. G. S. Young: *Concerning spaces in which every arc has two sides.*

Let S denote a connected, locally connected, complete metric space satisfying the following axiom: If AB is an arc and D is a domain containing $AB-(A+B)$, then D contains a connected domain which is separated by $AB-(A+B)$ into two connected domains, each having AB in its boundary. In this paper it is shown that if S is locally compact, it is a 2-manifold without boundary, which is closed if S is compact, and that if S is not locally compact, but satisfies certain "flatness" conditions, then it can be imbedded in a 2-manifold. A similar characterization and imbedding theorem is given for 2-manifolds with boundary. Several characterizations of the sphere are also given. (Received March 27, 1944.)

171. G. S. Young: *On continua whose links are non-intersecting.*

In this note, it is shown that if a compact metric continuum is not a simple link of itself and no two of its links intersect, then uncountably many are degenerate; also that the statement obtained by replacing the words "compact metric continuum" by "connected, locally connected, separable Moore space" is true. (Received March 27, 1944.)

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