of treating difference equations is established and can be applied when $b_{ij}(x) \equiv 0$, and also when $p(x)$ is replaced by 0 in (1). The method makes use of analytic implicit functions and a matrix transformation, and the case where $b_{ij}(x) \not\equiv 0$ is made to depend upon the case $b_{ij}(x) \equiv 0$. (Received February 1, 1944.)

149. Otto Szász: *On uniform convergence of trigonometric series.*

This paper contains generalizations of some theorems due to Chaundy and Jolliffe, to Hardy, and to the author. The following are some of the results. The trigonometric series $\sum_1^\infty b_n \sin nt$ is uniformly convergent if $\sum_n^{2n} |b_\nu - b_{\nu+1}| = O(n^{-1})$ and if the sequence nb_n is Abel summable to zero. The power series $\sum_1^\infty c_n z^n$ is uniformly convergent in $|z| \leqq 1$, if $\sum_n^{2n} |c_\nu - c_{\nu+1}| = O(n^{-1})$ and if $\sum c_n$ is Abel summable. The essential part of the proof concerns the point $z=1$, that is $t=0$; a device of the Tauberian type is employed. (Received March 18, 1944.)

150. F. A. Valentine: *Contractions in non-euclidean spaces.*

Let $f(x)$ be a function mapping a set S in a metric space M into a set S' in a metric space M', and suppose a contraction of the type $\|f(x_1), f(x_2)\|' \leqq \|x_1, x_2\|$ holds in S and S'. The existence of an extension of the range of definition of such a function so as to preserve a contraction depends upon M and M'. In this article the author shows *the extension exists when $M = M'$ is the n-dimensional hyperbolic* space. The proof used is applied to a metric space which includes both the hyperbolic and the spherical cases. Hence a unification of results is also obtained. (Received February 21, 1944.)

151. S. E. Warschawski: *On conformal mapping of nearly circular regions.*

Generalizing results of L. Bieberbach (Sitzungsberichte, Berliner Akademie, 1923) and of A. R. Marchenko (Bull. Acad. Sci. U.S.S.R. 1935), the author proves the following theorem: Let R be a simply connected region with the properties: (i) R contains the origin $w=0$ and its boundary lies in the ring $1 \leqq |w| \leqq 1+\epsilon$, ϵ being a fixed positive number; (ii) there exists a number $\eta \geqq \epsilon$ such that any two points P_1 and P_2 of R of distance less than ϵ can be connected in R by an arc of *diameter* less than η. If $w=f(z)$ maps the circle $|z|<1$ conformally onto R ($f(0)=0$, $f'(0)>0$) then, for all $|z|<1$, $|f(z)-z| \leqq B\epsilon \log(1/\epsilon)+4\eta$, where B is an absolute constant. Analogous results for the derivatives of the mapping function, such as the following, are established. Let C be a simple closed curve $\rho=\rho(\phi)$, $0 \leqq \phi \leqq 2\pi$ (ρ, ϕ polar coordinates), such that $1 \leqq \rho(\phi) \leqq 1+\epsilon$, $|\rho'/\rho| \leqq \epsilon$, and that $|\rho'(\phi_2)/\rho(\phi_2) - \rho'(\phi_1)/\rho(\phi_1)| \leqq \epsilon|\phi_2 - \phi_1|$, $0<\epsilon<1$. If $f(z)$ (normalized as above) maps $|z|<1$ onto the interior of R, then, for $|z| \leqq 1$, $(A(1+\epsilon^2)^{1/2})^{-1} \leqq |zf'(z)/f(z)| \leqq A(1+\epsilon^2)^{1/2}$ and $|f'(z)-1| \leqq 5(A\epsilon + A - 1)$, where $A = 4^\epsilon e^{\epsilon^2}$. (Received April 1, 1944.)

Applied Mathematics

152. Wilfred Kaplan and Max Dresden: *The mechanism of the condensation of gases.*

The criterion previously formulated (see abstract 49-5-158) for the condensation of a gas: namely, that condensation occurs at energy zero, when the topological structure of the energy surface changes, is further explored. It leads to a qualitative picture

of the whole process of condensation. The theory has now been compared with experiment. The critical temperatures of seven inert gases are predicted from the known critical volumes, and a satisfactory agreement with observed critical temperatures is found. (Received March 23, 1944.)

153. Morris Marden: *Axisymmetric harmonic vectors.*

Let $\Phi(x, \rho)$ and $\Psi(x, \rho)$, where $\rho^2 = y^2 + z^2$, be respectively the potential and Stoke's stream function in an axisymmetric flow and let i and m be the unit vectors along the x- and ρ-axis of the meridian plane. Then with the harmonic vector $H = \Phi i - \rho^{-1}\Psi m$ may be associated an analytic function $\phi(x+i\rho)$ of a complex variable as follows (cf. S. Bergman, Bull. Amer. Math. Soc. vol. 49 (1943) pp. 163–174): $H = (2\pi)^{-1} \cdot \int_0^{2\pi} \phi(x+i\rho \cos t)(i+ie^{it}m)dt$. This formula is used in the present paper to study the relation between the two-dimensional flow with complex potential $\phi(x+i\rho)$ and the three-dimensional axisymmetric flow with potential $H(x, \rho)$. The $\phi(u)$ corresponding to certain distributions of doublet sources or vortices is found to be the associate of the $H(x, \rho)$ corresponding to a circular source or vortex filament. The $\phi(u)$ corresponding to "Karman streets" of doublet sources or vortices is thus found to be the associate of an $H(x, \rho)$ corresponding to Karman streets of circular source or vortex filaments. Finally, the energy in an axisymmetric force field is expressed operationally in terms of the energy in the associate two-dimensional field. (Received March 25, 1944.)

154. A. D. Michal: *Physical models of some curved differential-geometric metric spaces of infinite dimensions.* II. *Vibrations of elastic beams and other elastic media as studies in geodesics.* Preliminary report.

This is a continuation of a previous paper (abstract 49-11-289). In this paper a special study is made of the infinite dimensional Riemannian geometries determined by conservative vibration systems of beams and other elastic media. (Received April 1, 1944.)

155. Ida Roettinger: *Certain integral transformations, with applications to boundary value problems.*

The transformations $S\{F(x)\} = \int_0^\pi F(x) \sin (n-1/2)x dx$ and $C\{F(x)\} = \int_0^\pi F(x) \cos (n-1/2)x dx$, $n = 1, 2, 3, \cdots$, where $F(x)$ is sectionally continuous on the interval $(0, \pi)$, are studied. Four convolution theorems are proved, analogous to those given by Kniess for the ordinary $\sin nx$ and $\cos nx$ transformations. (See H. Kniess, Math. Zeit. vol. 44 (1938) pp. 266–292.) By means of one of these theorems the solution of the heat conduction problem $U_t(x, t) - K(t)U_{xx}(x, t) + A(t)U(x, t) + Q(x, t) = 0$, $U_x(0, t) = B(t)$, $U(\pi, t) = C(t)$, $U(x, 0) = F(x)$ is expressed in terms of the solution of the problem $V_t(x, t) = V_{xx}(x, t)$, $V(0, t) = 0$, $V_x(\pi, t) = 0$, $V(x, 0) = 1$. Similar work has been done by Brown. (See H. K. Brown, Journal of Applied Physics vol. 14 (1943) pp. 609–618.) Furthermore the transformations $\Lambda\{F(x)\} = \int_{-1}^1 F(x) \sin \lambda_n(1+x)dx$ and $\bar{\Lambda}\{F(x)\} = \int_{-1}^1 F(x) \sin \lambda_n(1-x)dx$, where λ_n are the positive roots of $\tan 2\lambda = -\lambda/h$, $h > 0$, are studied and applied to the above heat equation with the boundary conditions $U(-1, t) = H(t)$, $U_x(1, t) + hU(1, t) = G(t)$, $U(x, 0) = F(x)$. Three convolution theorems are proved for the Λ-transformation. (Received March 16, 1944.)

156. J. A. Shohat: *Series expansions for the periodic solution of Van der Pol's equation and its frequency for all values of the parameter.*

If the parameter μ in Van der Pol's equation $d^2u/dt^2-\mu(1-u^2)du/dt+u=0$ is small, power series expansions for the periodic solution u (unique, save for time-translation) and its frequency v can be and have been given, say, by Lidstedt's method. In the present paper the author gives (for the first time, he believes) series expansion for u and v, valid for all values of μ—large and small. Numerical computation agrees quite well with known numerical results. (Received March 13, 1944.)

GEOMETRY

157. Reinhold Baer: *The fundamental theorems of elementary geometry. An axiomatic analysis.*

It is the object of this paper to evaluate the logical interdependence of certain fundamental theorems in elementary geometry. The paper deals with the theorems asserting the copunctuality of each of the following triplets of lines: medians, altitudes, perpendicular bisectors, and bisectors of angles; and the theorem stating that the locus of the points of equal distance from two different points is a line. The framework of our discussion is provided by a general affine plane in which we introduce just as many further relations as are needed for stating the investigated theorems. (Received March 22, 1944.)

158. P. O. Bell: *A study of surfaces by means of a system of differential equations of the first order.*

The projective differential geometry of a surface in ordinary space is studied by means of tetrads of surfaces whose corresponding points x_i ($i=0, 1, 2, 3$) are linearly independent. The general homogeneous coordinates of x_i satisfy a system of equations $\partial x_i/\partial u^\alpha = C_{h,i\alpha}x_h$, $\alpha=1, 2$, summed for $h=0, 1, 2, 3$. With the points x_i as vertices of a local reference tetrahedron an algebraic surface $a_{ij\ldots l}x^ix^j\cdots x^l=0$ is fixed as u^1, u^2 vary independently, if and only if the coefficients $a_{ij\ldots l}$ are proportional to the corresponding components of the covariant derivatives, of the aggregate of these coefficients, with respect to the connection $C_{h,i\alpha}$. Such conditions of immovability form the basis for a general theory of envelopes. Tetrads of surfaces are first investigated. The study of a surface S_0 is then undertaken by specializing the general theory. Auxiliary surfaces S_1, S_2, S_3 covariantly determined with respect to S_0 are selected so that the fundamental differential equations are as simple as possible and exhibit desirable properties of symmetry. Some differential invariants are characterized geometrically. When the asymptotic curves are parametric on one of the surfaces one of these invariants becomes the projective linear element and another becomes Fubini's element of projective arc length. (Received April 1, 1944.)

159. S. S. Chern: *Laplace transforms of a class of higher dimensional varieties in a projective space of n dimensions.*

In a projective space of n dimensions a class of r-dimensional varieties is defined, which form a natural generalization of the surfaces sustaining conjugate nets. These varieties are characterized by the property that the asymptotic net is an $(r-1)$-parameter linear system of cones whose base cones are linear spaces counted twice. (See E. Cartan, Bull. Soc. Math. France vol. 47 (1919) pp. 125–160.) This geometrical