271. Stefan Bergman: *Operators in the theory of partial differential equations and their application.* II.

Let $v(x, y)e^{i\theta(x,y)}$ denote the velocity vector of an irrotational steady flow of compressible fluid. Let $\zeta=\Lambda(v)+i\theta$, $\bar{\zeta}=\Lambda(v)-i\theta$, where $d\Lambda(v)/dv=[1-M^2]^{1/2}/v$, and $M=v/[d_0^2-(1/2)(k-1)v^2]^{1/2}$, d_0 and k being constants. Finally: let $E^*=1+t\zeta^{1/2}Q(\zeta, \bar{\zeta}, t\zeta^{1/2})$ where $Q(\zeta, \bar{\zeta}, p)$ is an (arbitrary) solution of $Q_{p\bar{\zeta}}+2p(Q_{\zeta\bar{\zeta}}+FQ)+2F=0$, and Q is supposed to be an odd function of p. Then $\psi(v, \theta)=Re\{\int_{-1}^{1}T(\zeta+\bar{\zeta})E^*(\zeta, \bar{\zeta}, t)f[(1/2)\zeta(1-t^2)]dt/(1-t^2)^{1/2}\}$ where f is an arbitrary analytic function of one complex variable is the stream function of a suitable subsonic flow, and the stream function of every flow can be represented in the above form. T and F are suitable functions of $(\zeta+\bar{\zeta})$. Using this result the author proves that various sets of particular solutions $\{p_\nu(v, \theta)\}$ are complete. The author indicates a method of determining the constants $a_\nu^{(n)}$ in $\psi_n=\sum_{\nu=1}^{n}a_\nu^{(n)}p_\nu$ in such a way that ψ_n approximates the flow in a channel which is given in the xy-plane (physical plane), provided that the image of the flow in the hodograph plane is schlicht. The method is a generalization of one given in Duke Mathematical Journal, vol. 6 (1940), p. 537. A similar procedure can be applied in the case of a supersonic flow. (Received July 28, 1942.)

272. Vladimir Morkovin: *On the deflection of anisotropic thin plates.*

Deflections w of an anisotropic plate (with one plane of elastic symmetry) bounded by an analytic curve C_0 are considered. The general solution of the differential equation for w is known to be expressible in terms of two analytic functions $f_1(z_1)$ and $f_2(z_2)$, where the complex variables z_1 and z_2 are related to the variable z_0 of the original plane by $z_k=p_kz_0+\bar{q}_k\bar{z}_0$, the constants p_k and q_k depending on the material of the plate. (See S. N. Lechnitzky, Journal of Applied Mathematics and Mechanics, (n. s.), vol. 2 (1939), pp. 181–210.) Transformations $z_k=\omega_k(\zeta_k)$ are found which make any point on C_0 correspond to points $e^{i\theta}$ on the circumferences γ_k of unit radii in new ζ_1 and ζ_2 planes having the same polar angle θ, and which are conformal in some neighborhoods of γ_k. Then the functions $\phi_k(\zeta_k)\equiv f_k(z_k)$ can be determined from the two given boundary conditions if these are expressed in terms of $e^{i\theta}$. A detailed solution illustrating this general procedure is carried out in the case of a clamped elliptic plate with polynomial loading. (Received July 31, 1942.)

Geometry

273. H. S. M. Coxeter: *A geometrical background for the description of de Sitter's world.*

This paper begins with an elementary treatment of the process by which an elliptic or hyperbolic metric in the plane at infinity of affine space induces a Euclidean or Minkowskian metric in the whole space. The various kinds of sphere are defined, and are seen to provide models for non-Euclidean planes, including the "exterior-hyperbolic" plane which is a two-dimensional de Sitter's world. (See Eddington, *The Mathematical Theory of Relativity*, 1924, p. 165.) Then comes a simple proof of Study's theorem to the effect that one side of a triangle is greater than the sum of the other two, and finally a discussion of some cosmological paradoxes. (Received July 31, 1942.)

274. J. J. DeCicco: *New proofs of the theorems of Beltrami and Kasner on linear families.*

Here new proofs of the theorems of Beltrami and Kasner on linear families of curves are submitted. Beltrami's result is that a surface S may be mapped upon a plane π so that its geodesics correspond to straight lines if and only if S is of constant Gaussian curvature. Kasner's result states that a complete system of isogonal trajectories of a simple family of curves F is linear if and only if F is isothermal. From this work is also deduced another theorem of Kasner which is that a surface S can possess exactly ∞^2 isothermal families of geodesics if and only if S is of constant Gaussian curvature. A simple uniform method is used in all the proofs. (Received July 30, 1942.)

275. J. J. DeCicco: *The application of turbine geometry to the inverse problem of dynamics.*

Kasner has discussed analytically the inverse problem of dynamics. (See *Differential-Geometric Aspects of Dynamics*, American Mathematical Society Colloquium Publications, vol. 3, 1913.) In this paper a synthetic solution of this is given by use of turbine geometry, and the whirl-motion group G_6. In the course of this development, a new type of element-series is introduced, which will be called a limaçon series of the second kind. This is different from the limaçon series of the first kind, which was discussed in *The geometry of fields of lineal elements*, Transactions of this Society, vol. 47 (1940), pp. 207–229. The indeterminate case is completely explained. (Received July 30, 1942.)

276. B. H. Gere and David Zupnik: *On the construction of curves of constant width.*

General properties of a curve of constant width, which is a curve such that the distance between parallel tangents is a constant, were given by Barbier (Journal de Mathématiques Pure et Appliquées, (2), vol. 5 (1860), pp. 273–286). The purpose of this paper is to give a general method for constructing such curves. An evolute star, which is a configuration of n curvilinear sides, n odd, is first defined. It is shown how, with any such star as evolute, one and only one family of parallel curves of constant width may be constructed. A necessary and sufficient condition in terms of the lengths of the sides of the star is found for the construction of a curve of specified constant width. Finally, the same method of construction is shown applicable to degenerate stars, giving curves which contain circular arcs. (Received July 20, 1942.)

277. Edward Kasner: *Differential equations of the type:* $y''' = Gy'' + Hy''^2$.

Certain problems in geometry and physics give rise to a general third order differential of the type (G): $y''' = Gy'' + Hy''^2$, where G and H are functions of (x, y, y') only. The family of ∞^3 integral curves of any such equation may be characterized as follows. For any curve of the family containing an arbitrary lineal element E, construct the osculating parabola at E. The foci of the ∞^1 parabolas so obtained will describe a circle through the point of E. An equivalent property is that the directrices will form a pencil of lines. The following eight problems are discussed: (1) the ∞^3 trajectories of a given field of force; (2) the complete system of velocity curves; (3) the systems S_k; (4) the general catenaries; (5) the ∞^3 brachistochrones in a conservative field; (6) sectional families; (7) curvature trajectories, and (8) additive-multiplicative trajectories. The type (G) was first encountered in the study of dynamical trajectories (*Differential-Geometric Aspects of Dynamics*, American Mathematical Society Colloquium Publications, vol. 3, 1913; *The trajectories of dynamics*, Transactions of this Society, vol. 7 (1906), pp. 401–424). All eight problems are distinct but lead to species of this type. Recently Terracini has given a new projective characterization of (G). (Received July 7, 1942.)

278. Edward Kasner: *Differential equations of the type*: $y^{\text{iv}}=Ay'''^2+By'''+C$.

In this paper are discussed the various problems connected with general fourth order equations of the type (Q): $y^{\text{iv}}=Ay'''^2+By'''+C$, where A, B, C are functions of (x, y, y', y'') only. The systems of ∞^4 integral curves of any such equation may be characterized as follows. For any curve of the system containing an arbitrary curvature element E_2, construct the osculating conic at E_2. The ∞^1 conics so obtained have their centers on a conic which contains the lineal element E_1 of E_2. The following seven problems are discussed: (1) the sets of ∞^4 curves characterized by linear families of osculating conics; (2) the systems of extremals in a calculus of variations problem; (3) the systems of curves in a field of force for which the pressure is proportional to the normal component of the force; (4) the system of curves in a field of force for which the pressure is constant; (5) the system of trajectories of a particle of special variable mass in a positional field of force; (6) the orthogonal trajectories of the space-time curves of a field of force which depends not only on the position but also upon the time, and (7) the complete set of the rate of variation of curvature trajectories. These seven species are distinct but all lead to (Q), which the author first encountered in a study of extremals (this Bulletin, vol. 14 (1908), pp. 461–465). (Received July 7, 1942.)

279. Edward Kasner: *The inverse problem of dynamics*.

In any given positional field of force of the plane, there are ∞^3 trajectories. Conversely, if the totality of trajectories is given, the field of force is determined except for a constant factor. See *Differential-Geometric Aspects of Dynamics*, American Mathematical Society Colloquium Publications, vol. 3, 1913, for a discussion of "the geometric exploration of a field." The essential problem discussed in this paper is to find the direction of the force at a given point 0 when only four trajectories through 0 are given. The complete analytic solution is treated. Usually the solution is unique but there arises an important indeterminate case. (Received July 30, 1942.)

280. Edward Kasner and J. J. DeCicco: *Transformation theory of isogonal trajectories of isothermal families*.

In the paper, *Lineal element transformations which preserve the isothermal character* (Proceedings of the National Academy of Sciences, vol. 27 (1941), pp. 406–409), Kasner determined the complete group of lineal element trasnformations of the plane which send any isothermal family of curves into an isothermal family. In this new work, it is the purpose of the authors to extend this result to transformations of second differential elements. The total group of curvature element transformations carrying any complete system of isogonal trajectories of isothermal families into any other such system is found in explicit form. The only contact transformations of Kasner's group and this new group are the conformal maps. The doubly-infinite systems of curves mentioned above are identical with the conformal images of the ∞^2 straight lines, and thus may properly be described as "conformal rectilinear wexes." The content of the new group is $2\infty^{6f(1)}$, as contrasted with the content $2\infty^1+4f(1)$ of the old group. (Received June 19, 1942.)

281. Domina E. Spencer: *Geometric figures in affine space*.

A study of the figures in affine space has been undertaken in order to establish the place of Study's "Geometrie der Dynamen" in the structure of geometry. Geometric figures have been considered whose tensor representations are: 1. contravariant alternating tensors of valence two; 2. covariant alternating tensors of valence two; 3. mixed tensors of valence two. Among this set are found the affine ancestors of all of the Study figures. Thus the foundation is laid for the tensor interpretation of all of the figures of the "Geometrie der Dynamen." (Received July 30, 1942.)

Statistics and Probability

282. L. A. Aroian: *The relationship of Fisher's z distribution to Student's t distribution.*

For n_1 and n_2 sufficiently large, $W=(1/\beta)(N/(N+1))^{1/2}z$ is distributed as Student's t with N degrees of freedom, $N=n_1+n_2-1$, $\beta^2=(1/2)(1/n_1+1/n_2)$. If the level of significance is α for Student's distribution, the level of significance for z will be $(N/(N+1))^{1/2}\alpha e^{\beta^2/3-5/12N}<\alpha$. As a corollary it follows that the distribution of z approaches normality, n_1, $n_2\rightarrow\infty$, with mean zero and variance β^2. This simplifies a previous proof of the author. Application of this result is made to finding levels of significance of the z distribution. On the whole R. A. Fisher's formulas for finding such levels, n_1 and n_2 large as modified by W. G. Cochran, are superior. The formulas of Fisher-Cochran are compared with the recent formula of E. Paulson. (Received August 1, 1942.)

283. E. J. Gumbel: *Graphical controls based on serial numbers.*

The index m of the observed value x_m $(m=1, 2, \cdots, n)$ is called its serial number (or rank). A value x of the continuous statistical variable defined by a probability $W(x)=\lambda$ is called a grade (for example, the median). The coordination of serial numbers with grades furnishes two graphical methods for comparing the observations and the theory, namely the equiprobability test based on $m=n\lambda$, and the return periods based on $m=n\lambda+1/2$. From the distribution of the mth value, determine the most probable serial number $\tilde{m}=n\lambda+\Delta$, where Δ depends upon the distribution. For a symmetrical distribution, the corrections for two grades defined by λ and $1-\lambda$, are $\Delta(1-\lambda)=-\Delta(\lambda)$. For an asymmetrical distribution, calculate the most probable serial number of the mode considered as an mth value. Thus the mode is obtained from the observations, but it is not the most precise mth value. If m is of the order $n/2$ the distribution of the mth value converges towards a normal distribution with a standard deviation $s(x)=(W(x)(1-W(x)))^{1/2}/(w(x)n^{1/2})$. The intervals $x\mp s(x)$ give controls for the equiprobability test, the step function and the return periods. Besides, the standard error of the mth value leads to the precision of a constant obtained from a grade. (Received July 20, 1942.)

284. Mark Kac: *On the average number of roots of a random algebraic equation.*

Let $X_0+X_1x+\cdots+X_{n-1}x^{n-1}=0$ be an algebraic equation whose coefficients $X_0, \cdots, X_{n-1}$ are independent random variables having the same normal distribution with density $\pi^{-1/2}\exp(-u^2)$. If $N_n=N_n(X_0, \cdots, X_{n-1})$ denotes the number of real roots of the equation then the average number of roots (mathematical expectation of N_n) is asymptotically equal to $2\pi^{-1}\log n$. Moreover, for $n\geqq 2$ the mathematical expectation of N_n is not more than $2\pi^{-1}\log n+14\pi^{-1}$. This is an improvement of a result of Littlewood and Offord (Journal of the London Mathematical Society,