philosophical themes such as, *Whitehead's theory of value* and *Whitehead's idea of God.*

The book as a whole is well designed, and the print is good. There is an occasional typographical error such as formula (12) on page 150 which should read,

$$[\hat{\alpha} \sim (\alpha \in \alpha) \in \hat{\alpha} \sim (\alpha \in \alpha)] \equiv \sim [\hat{\alpha} \sim (\alpha \in \alpha) \in \hat{\alpha} \sim (\alpha \in \alpha)].$$

A. R. TURQUETTE

Introduction to Logic and to the Methodology of Deductive Sciences. By Alfred Tarski. Enlarged and Revised Edition. New York, Oxford University Press, 1941. 18+239 pp. $2.75.

This is an amplified and revised version of a book which first appeared in Polish in 1936, and was translated into German in 1937. The intention of the original book was to give an elementary but clear account of the concepts of modern mathematical logic for the benefit of readers interested in mathematics but with no technical knowledge of it beyond that possessed by a well trained college freshman. In the English version various additions have been made to make the work more suitable as a textbook for college courses.

This book and its preceding editions have been already reviewed in several places; in particular the German version was reviewed in this Bulletin (vol. 44, p. 317) by Quine. For a considerable list of other reviews see the indexes to volumes 4 (1939) and 6 (1941) of the Journal of Symbolic Logic—on pp. 193 and 187 respectively—; to the lists there given should be added the review by Frink in Mathematical Reviews, vol. 2 (1941), p. 209. In view of this fact it is superfluous for the present reviewer to do more than summarize the general purport of these reviews and to add to the criticisms certain amplifications of his own.

All reviewers, including the present one, are agreed that this is a work of exceptional merit. For the purpose for which it was originally designed it is a masterpiece of exposition. Whether the patching which the book has received to convert it into a textbook will succeed in that endeavor is doubtful—for some there will not be enough technique and for others not enough application to extra-mathematical domains—; but the value of the book for the independent reader is enhanced thereby. The exercises at the ends of the chapters are an excellent feature. For the seasoned mathematician the book is, perhaps, too easy, and it certainly does not give an adequate idea of the difficulty of some logical problems; nevertheless it contains material of interest and value. Within the limitations imposed by its objectives it is the most accurate and perspicuous survey of its subject which now exists.

A defect which has been noticed by several reviewers is an unwarranted departure from accepted terminology. This is especially patent in the case of the algebra of propositions. Here the book avoids the word 'proposition' altogether; in the place of it and its derivatives we find such words as 'sentence,' 'sentential calculus,' and so on. There are two circumstances which probably have a bearing on this choice of terminology. In the first place in the German language there is no distinction analogous to that between the English words 'proposition' and 'sentence'; there are, to be sure, two words, 'Satz' and 'Aussage,' but the distinction between them is not parallel. Now those who prefer the 'sentence' terminology have generally come under the influence of the Vienna Circle, which originated, of course, in a German-speaking community; and it is likely that some of these persons do not sense the violence they are doing to the linguistic instincts of those for whom the English language is native. (It is said that Carnap, since he has lived in America, has become more sympathetic to the use of 'proposition.') In the second place the word 'proposition' seems vague and has for some a metaphysical connotation; these propositions must be mysterious beings indeed, since we have no exact criteria for identity amongst them; moreover they are unnecessary.

In defense of the more usual terminology, reply may be made as follows. In the first place it is purely a mistake in translation to foist upon the English language an incongruous Germanism. Moreover, the notion of sentence is just as vague as that of proposition. Indeed consider the following lines of type (suggested by Church's review—see the list above cited):

(1) $2+3=5$
(2) The sum of 2 and 3 is 5.
(3) The sum of 2 and 3 is 5.
(4) The sum of 2 and 3 is 5.
(5) *The sum of 2 and 3 is 5.*
(6) Die Summe von 2 und 3 ist 5.
(7) Die Summe von 2 und 3 iſt 5.
(8) $3+2=5$
(9) $2+3\neq 5$

How many sentences are there? For Lesniewski presumably there would be $9n$, where n is the number of copies of this magazine; for a typographer, 8; for Quine, who says he regards a sentence as a shape, 7; for a grammarian 5; for a German logician, understanding 'sentence' as translation of 'Satz,' there might be 3. (Carnap's usage in the *Logical Syntax* is distinct from any of the first four; for when Countess Zeppelin translated his book into English she was compelled to translate inside quotation marks in order to preserve the meaning which Carnap wished to convey, which translation would have been improper if any of the first four senses had been intended.) Moreover the use of 'proposition' does not connote a belief in mysterious entities running around the universe; one can simply understand a proposition as a sentence in the above logician's sense. Finally the vagueness inherent in both the words 'proposition' and 'sentence' is irrevelevant since in logic we have no concern with equality of propositions but only their equipollence. In short the replacement of 'proposition' by 'sentence' serves no useful purpose and introduces, rather than eliminates, confusion. This is the opinion of every mathematician, so far as the reviewer knows, who has commented on the subject; Quine, who would doubtless disagree, is a philosopher. It should be noted that there is no departure from accepted usage in the German version; there 'Satz' and 'Aussagenkalkül' are used in their usual senses.

There is a related but distinct criticism in regard to the interpretation of the propositional algebra. Tarski uses the syntactical interpretation, whereby the variables 'p,' 'q' are to be replaced by sentences and the connectives ('$\vee$,' '$\rightarrow$,' '$\sim$') are conjunctions. Another interpretation is possible, namely: where these variables are replaceable by nouns, that is, noun clauses such as 'that 2 precedes 3.' On the first interpretation '$p\rightarrow q$' gives rise to the sentence

if 2 *precedes* 3, *then* 3 *follows* 2;

on the second to the complex clause

that (*that* 2 *precedes* 3) *implies* (*that* 3 *follows* 2),

which, when preceded by the assertion sign (or some similar indication) becomes the sentence

that 2 *precedes* 3 *implies that* 3 *follows* 2.

Either interpretation can be used, the second having the advantage of making the algebra have characteristics more similar to those of mathematical theories in general. As Quine points out, Tarski vacillates between these two.

A final criticism is that the first chapter, on variables, could profitably be completed by considering the types of bound variables that occur in the infinitesimal calculus—for example, in

$$\int_1^2 x^2dx, \ \frac{d}{dx}\, x^2.$$

Such an addition might lead up to an explanation of the Church λ-quantifier, in terms of which, in connection with constants, all other quantifiers can be defined.

HASKELL B. CURRY

Two-Dimensional Potential Problems Connected with Rectilinear Boundaries. By B. R. Seth. Allahabad, India, The Allahabad Law Journal Press, 1939. 118 pp.

This attractive little volume grew out of a course of lectures delivered by Dr. Seth at the University of Lucknow in 1939. It deals primarily with the application of the Schwarz-Christoffel transformation in the solution of several problems and potential theory and related fields of mathematical physics. Among the problems discussed are special cases of the problem of torsion of a long prism as well as the Saint-Venant flexure problem, and problems of ideal fluid flow around prisms.

The author restricts himself to rectilinear boundaries for which the Schwarz-Christoffel transformation allows mapping on a half-plane. By confining himself to rectangular regions, special triangular regions such as the equilateral triangle, the 90°, 60°, 30° triangles and other similar special regions, he is able to express the mapping and the solutions of the problems for them in terms of the classical elliptic functions. Among other regions considered is the "angle-iron," the region on the outside of a rectangle, and the L-section.

The book will be welcomed by workers in this field of mathematical physics, as well as by mathematicians who are interested in application of elliptic functions.

H. PORITSKY

Operational Methods in Applied Mathematics. By H. S. Carslaw and J. C. Jaeger. Oxford, Clarendon Press, 1941. 16+264 pp. $5.00.

In May 1815 Simeon Denis Poisson read before the Paris Academy a memoir on the distribution of heat in solid bodies. Extracts from this memoir were at once published in the Journal de Physique and in the Bulletin de la Société Philomathique. The memoir was subsequently enlarged and became, perhaps, one of Poisson's favorites because in May 1821 the work was printed and distributed privately two years before its final publication in Journal de l'École Polytechnique, vol. 12, no. 19, pp. 1–162. Poisson here made, I think, the first use of the method of the inverse Laplace transformation. In an attempt to find the distribution of temperature in a uniform rod radiating at its ends he was led to two linear functional differential equa-