212. C. B. Tompkins: *Local imbedding of Riemannian spaces.*

The paper gives a proof of Janet's theorem on local imbedding of an n-dimensional Riemannian space in euclidean space of $n(n+1)/2$ dimensions. The proof is by induction on the number of dimensions, and the induction is made possible by strengthening the theorem slightly to state that the Riemannian space can be imbedded so that its tangent plane and the vectors obtained by differentiating the imbedding functions twice with respect to a set of $(n-1)$ of the n parameters of the Riemannian space together span the euclidean space. The proof involves an existence theorem in differential equations, a warping process somewhat analogous to the process of rolling a plane into a cylinder, and an algebraic lemma which is used to show that the warping process will furnish the independence of vectors required in the strengthened theorem of the inductive proof. (Received March 20, 1942.)

213. B. J. Topel and P. M. Pepper: *Imbedding theorems under weakened hypotheses.*

The congruent imbedding of a semi-metric space into E_n, the euclidean n-space, as implied by the imbeddability of as few as possible of its $(n+2)$-tuples is considered. For S, consisting of $n+4$ or more points, to be imbeddable into E_n, but not E_{n-1}, it is necessary and sufficient that S contain a certain nucleus S' and that all $(n+2)$-tuples containing at least n points of S' be imbeddable. Sufficient conditions are derived for the imbeddability of S when an upper bound is placed on the number of non-mapping $(n+2)$-tuples with no restriction on their distribution in S. These theorems sharpen Menger's quasi-congruence theorem. Necessary and sufficient conditions are determined for the imbeddability of a semi-metric space into E_n. Similar theorems are considered for imbeddability into an element of any congruence system. The structure of non-mapping sets with the minimum number of non-mapping $(n+2)$-tuples is studied. (Received March 23, 1942.)

214. J. E. Wilkins: *A characterization of the quadric of Wilczynski.*

The quadric of Wilczynski at a point P of a non-ruled surface S in projective three-dimensional space is characterized as the unique quadric having second-order contact with S at P (therefore intersecting S in a curve with a triple point at P), the three triple-point tangents being the tangents of Darboux, and the three triple-point osculating planes having a line in common. This line is found to be the canonical line of the first kind with $k=5/12$. (Received March 12, 1942.)

Logic and Foundations

215. S. C. Kleene: *On the forms of the predicates in the theory of constructive ordinals.* Preliminary report.

In the system S_3 of notation for ordinal numbers (Journal of Symbolic Logic, vol. 3 (1938), p. 155), the class O of the natural numbers which represent ordinals, and the partial ordering relation $<_o$ between such numbers, were defined by a transfinite induction. It is now shown that the predicates $a\in O$ and $a<_o b$ are expressible explicitly in the respective forms $(x)(Ey)R(a, x, y)$ and $(x)(Ey)S(a, b, x, y)$ where R and S are primitive recursive predicates. The result can be used to exhibit the incompleteness of ordinal logics by a method presented previously. (See abstract 46-11-464. Erratum: for "for." read "for all.") The proof illustrates a technique to which recourse may be had generally in attempts to reduce inductive definitions to explicit definitions in terms of recursive predicates and quantifiers. (Received March 3, 1942.)

Statistics and Probability

216. J. H. Bushey: *The distribution function of the mean under the type α hypothesis.*

An orthogonal expansion (type α series) with the Pearson type III function as weight function is obtained in a form suitable to utilize the tables of Salvosa in representing population frequencies. The expansion differs in certain respects from that of Romanovsky. The distribution function of the mean is obtained for samples of n drawn at random from a population represented by the type α series. In special cases this distribution function reduces to that obtained for the Charlier type A series by Baker and to that obtained by Church for the type III function. (Received March 6, 1942.)

217. J. H. Bushey: *The distribution function of the sample total under the type β hypothesis.*

The orthogonal polynomials $\phi_n(x)$ are defined by the weight function $p(x) = C_{s,x}p^x(1-p)^{s-x}$, $(x=0, 1, 2, \cdots, s)$ and the orthogonal relation $\sum_{x=0}^{s} p(x) \cdot \phi_m(x)\phi_n(x) = 0$, $m \neq n$, or $=1$, $m=n$. The orthogonal expansion (type β series) $f(x) = \sum_{i=0}^{k} A_i p(x)\phi_i{}'x)$ may be used as a statistical hypothesis. The Charlier type A series is a special case of both the type β and the type α series (the type α series is reported in another abstract). Another special case of the type β series is the Charlier type B series with the Poisson weight function $p(x) = (e^{-sp}(sp)^x)/x!$. The distribution function for the total $z = n\bar{x}$ for samples of n drawn at random from a population represented by a type β series is obtained. This result includes, as special cases, the distribution function of z for the Charlier type B series and that obtained by Baker for the Charlier type A series. (Received March 6, 1942.)

218. J. H. Bushey: *The products of certain discrete and continuous orthogonal polynomials.*

The discrete orthogonal polynomials $\phi_n(x)$, $(x=0, 1, 2, \cdots, s)$, are defined by the weight function $p(x) = C_{s,x} \cdot p^x(1-p)^{s-x}$ and the orthogonal relation $\sum_{x=0}^{s} p(x) \cdot \phi_m(x)\phi_n(x) = 0$, $m \neq n$, or $=1$, $m=n$. The polynomials $\phi_n(x)$ are closely related to the continuous polynomials of Jacobi, Hermite, and Laguerre and have applications in statistics. The product $\phi_m(x) \cdot \phi_n(x)$ is developed in terms of the polynomials $\phi_i(x)$ for $p=q=1/2$ (symmetric polynomials). This development permits the evaluation of the sum $\sum_{x=0}^{s} p(x)\phi_m(x)\phi_n(x)\phi_r(x)$. The corresponding results of Feldheim for Hermite polynomials follow as special cases. (Received March 6, 1942.)

Topology

219. E. G. Begle (National Research Fellow): *Intersections of absolute retracts.*

Aronszajn and Borsuk have shown (Fundamenta Mathematicae, vol. 18 (1932) pp. 193–197) that if A and B are compact metric spaces such that their intersection, $A \cap B$, is an absolute retract, then their sum, $A \cup B$, is an absolute retract if and only