ment of this theory is use of the relationship between V_m and the symmetric group of degree m. For k of characteristic p the classical theory is valid only for $m<p$. In the present paper the decomposition of V_m is obtained for $p\leqq m<2p$. Further progress on the decomposition problem will likely have to await the development of the theory of modular representations of the symmetric groups of degree $m\geqq 2p$; the measure of difficulty here being the power of p which divides $m!$. (Received March 26, 1942.)

167. Bernard Vinograde: *Split rings and their representation theory.*

Suppose a ring R, with minimum condition on left ideals, can be split into a direct sum, $R=R^*+N$, of a semi-simple ring and the radical. This splitting property of R is equivalent to the existence of a certain set of noncommutative fields in R. Let R have a unit, and let V be a commutative group possessing R in its left operator domain and a finite composition series with respect to R. Then V is a direct sum of modules, each over one of the noncommutative fields. Thus V gives rise to a composite representation module for its homomorphism ring. In the case $V=R$ the resulting representation of R displays the essential structure of R^* and N, and may be considered a natural extension of the Wedderburn theorem for simple rings to the class of split rings with unit and finite composition series. (Received March 20, 1942.)

168. R. W. Wagner: *Expressibility relations for bilinear operations.* Preliminary report.

If xy is a multiplication operation for a vector space over a field, an expression like $(a((xb)(cy)))d$ is also a multiplication for the vector space. The operation x^*y is called simply expressible in the operation xy if x^*y is a finite iteration of the original operation. The operation x^*y is called strongly expressible if it is a linear combination of weakly expressible operations. Necessary and sufficient conditions are found for both kinds of expressibility. These conditions involve the enveloping algebra of the algebra formed by the original multiplication. Mutual expressibility, both simple and strong, is an equivalence relation and differs from isotopy. Reducibility is a property common to mutually expressible algebras. Associativity, commutativity, and the existence of nilpotents are properties which need not be preserved. (Received March 18, 1942.)

Analysis

169. Dorothy L. Bernstein and S. M. Ulam: *On the problem of completely additive measure in classes of sets with a general equivalence relation.*

The problem of finding a necessary and sufficient condition for the existence of a finitely additive measure in a class of sets, with the property that equivalent sets have equal measure, has been solved by Tarski (Fundamenta Mathematicae, vol. 31 (1938), pp. 47–66). This paper considers the existence of a completely additive measure in a class K which is a Borel field over a given sequence $\{A_n\}$ of sets and which has the property that $m(X)=m(Y)$ if X is equivalent to Y. Equivalence is defined in the most general sense: Given a division of the sets of K into disjoint classes $A_1, A_2, A_3, \cdots$, sets X and Y are called equivalent if they belong to the same class A_α. It should be noted that even if no requirement for equivalence is made, it is not always possible to find a measure in a Borel field over a given sequence of sets. A necessary and sufficient condition is obtained which involves the characteristic set of real numbers formed from the sequence $\{A_n\}$ and the existence of a homeomorphism of this set with a set of positive outer Lebesgue measure, in which intervals corresponding to equivalent sets have equal outer measure. (Received March 20, 1942.)

170. Lipman Bers: *A property of harmonic functions.*

Suppose that $u(z)=(x+iy)$ is a non-negative harmonic function defined in $|z|<1$. Consider the upper and lower limits, $L(\theta)$ and $l(\theta)$, of $u(z)$ as z goes toward the point 1 along a simple Jordan curve which possesses at 1 a definite direction forming the angle θ, $|\theta|<\pi/2$, with the positive x-direction. (These limits depend only upon θ.) $L(\theta)$ and $-l(\theta)$ are uniformly continuous and convex functions. If $-\pi/2\leqq\theta_1<\theta<\theta_2 \leqq\pi/2$, $l(\theta)\leqq\mu L(\theta_1)+(1-\mu)l(\theta_2)\leqq L(\theta)$ and $l(\theta)\leqq\mu l(\theta_1)+(1-\mu)L(\theta_2)\leqq L(\theta)$ where $\mu=(\theta_2-\theta)/(\theta_2-\theta_1)$. If $u(z)$ possesses at 1 a limit in one direction, that is, if $L(\theta_0) =l(\theta_0)$, $L(\theta)$ and $l(\theta)$ are linear functions in the intervals $-\pi/2\leqq\theta\leqq\theta_0$ and $\theta_0\leqq\theta\leqq\pi/2$. If $u(z)$ possesses at 1 limits in two different directions, $u(z)$ possesses at 1 a limit in any direction and $L(\theta)=l(\theta)$ is a linear function. If $u(z)$ is bounded and its Dirichlet integral finite, $L(\theta)$ and $l(\theta)$ are equal to the upper and lower limits of $\tau^{-1}\{\mu[V(\tau) -V(0)]+(1-\mu)[V(0)-V(-\tau)]\}$, $\tau\rightarrow+0$, where $\mu=(\pi+2\theta)/2\pi$ and $V(\tau)$ is the indefinite integral of $U(\tau)=u(e^{i\tau})$. (Received March 28, 1942.)

171. F. G. Dressel: *The fundamental solution of the parabolic equation.* II.

The existence of a fundamental solution in an unbounded region for the parabolic equation with variable coefficients is proved in this paper. In addition, several properties of this fundamental solution are demonstrated. (Received February 5, 1942.)

172. E. D. Hellinger and H. S. Wall: *Contributions to the analytic theory of continued fractions.*

The theory of (1) $1/(b_1+z)-a_1^2/(b_2+z)-a_2^2/(b_3+z)-\cdots$, as developed by Stieltjes, Hamburger, Hellinger, R. Nevanlinna, and others is extended to the case $a_n\neq 0$ real and $\mathfrak{J}(b_n)\geqq 0$. The nth approximant lies upon a circle $K_n(z)$ contained in $K_{n-1}(z)$, $(\mathfrak{J}(z)>0)$. *Case* I. For $\mathfrak{J}(z)>0$ the circles have only one point in common and (1) converges; the associated J-form which, in contrast with former extensions of this theory, is not hermitian, has a unique resolvent bounded on an "ellipsoid" E. *Case* II. Otherwise, (1) converges for all z (to a meromorphic function of z) if and only if it converges for $z=0$; the J-form has infinitely many resolvents completely continuous on E. The associated power series of (1) represents $f(z)$ asymptotically in $\epsilon\leqq\arg z\leqq\pi-\epsilon$, $\epsilon>0$, if and only if for every n there exists an M_n such that $f(z)$ lies in $K_n(z)$ for z in the sector and $\mathfrak{J}(z)>M_n$. If for $\mathfrak{J}(z)>0$ the value of an analytic function $f(z)$ is in $K_n(z)$ for every n, then $f(z)=\int_{-\infty}^{+\infty}d\phi(u)/(z+u)$, $\phi(u)$ real, bounded, monotone non-decreasing; and (1) represents a function of this form whenever it converges. The moments $\int_{-\infty}^{+\infty}u^n d\phi(u)$ do not in general exist. (Received March 6, 1942.)

173. Fritz John: *The character of solutions of linear partial differential equations.*

Let $L(u)=0$ denote a linear partial differential equation with analytic coefficients for a function $u(x_1,\cdots,x_n)$ (order N or "type" of equation as well as the number n of independent variables are arbitrary). Characteristic directions with respect to L at a point may be defined in the usual manner. Let there be given an analytic k-

parametric family of r-dimensional manifolds $M_{\alpha_1\cdots\alpha_k}$ in $x_1 \cdots x_n$-space ($r<n$), such that (a) all $M_{\alpha_1\cdots\alpha_k}$ have the same boundary, and (b) none of their normals is characteristic. Let $d\omega$ be any analytic element of mass on the $M_{\alpha_1\cdots\alpha_k}$. The main result of this paper states that $\int u d\omega$ over $M_{\alpha_1\cdots\alpha_k}$ will be an analytic function of $\alpha_1, \cdots, \alpha_k$, whenever u is any solution of class C^N of the equation $L(u)=0$. The analyticity of solutions of linear elliptic equations with analytic coefficients is a special case. It also follows that the values of u cannot be prescribed arbitrarily on any analytic manifold in $x_1 \cdots x_n$-space, which contains a sub-manifold M of lower dimension, which has no characteristic normals. (Received March 12, 1942.)

174. M. L. Kales: *Tauberian theorems.*

Let $T_{m,n} \geqq 0, \sum_{n=0}^{\infty} T_{m,n} \equiv 1$, and $\{A_n\}$ be a sequence of real numbers. Under fairly general conditions it is shown that if $\sum_{n=0}^{\infty} T_{m,n}A_n = O(1)$ as $m \to \infty$, and $A_m - A_n > -\psi(m, n)$ (where the function $\psi(m, n)$ is defined in the statement of the theorem) then $A_n = O(1)$ as $n \to \infty$. This result is then applied to prove the following theorem: Let $\lim_{x\to\infty} g''(x) = \alpha (0 < \alpha \leqq \infty)$. Let $T_{m,n} = (1/\sum_{k=0}^{\infty} \exp\{-g(k)+kg'(m)\}) \exp\{-g(n)+ng'(m)\}$. If $\lim_{m\to\infty} \sum_{n=0}^{\infty} T_{m,n}A_n = A$, and if $A_n - A_{n-1} > -K \exp\{g(n)-g(n-1)-g'(n-1)\}$, then $\lim_{n\to\infty} A_n = A$. The proof of this theorem for the case where α is infinite follows immediately from the first theorem. When α is finite the problem can be reduced to one where an application of one of Wiener's general Tauberian theorems is possible. (Received March 18, 1942.)

175. Meyer Karlin: *Characterization of the extremals of a variation problem of higher order in the plane.* Preliminary report.

A variation problem of second order in the plane, $\int \phi(x, y, y', y'')dx = \min$, gives rise to a quadruply infinite system of curves for its extremals. E. Kasner (this Bulletin, vol. 13 (1907), pp. 289–292) found their differential equation to be of the form: $y^{\mathrm{IV}} = Ay'''^2 + By''' + C$, where A, B, C involve x, y, y', y''. With the help of a theorem by A. Hirsch (Mathematische Annalen, vol. 49 (1897), pp. 49–72), the author develops a set of necessary and sufficient conditions that A, B, C must satisfy in order that the curves obeying the above differential equation be identifiable with the totality of extremals connected with a variation problem of second order in the plane. When the conditions are fulfilled, the solution is unique up to an additive arbitrary total derivative with respect to x of a function of x, y, y'. Examples of extremal and nonextremal families of the above quadratic type are given. The paper also contains some necessary conditions for, as well as geometric characterizations of, the ∞^6 and ∞^{2n} curves constituting, respectively, the totality of extremals of variation problems of the third and nth orders in the plane. (Received March 6, 1942.)

176. A. N. Lowan, H. E. Salzer and Abraham Hillman: *Coefficients of differences in the expansion of derivatives in terms of advancing differences.*

The exact values of the coefficients $A_{n,r}$ of the rth differences in Markoff's formula for the nth derivative in terms of advancing differences were computed for $n=1, 2, \cdots, 20$ and $r=n, n+1, n+2, \cdots, n+20$ by the Mathematical Tables Project, Work Projects Administration, New York City. The coefficients are defined by $A_{n,r} = (n/r(r-n)!)\, B_{r-n}^{n}$ where B_{r-n}^{r} is the $(r-n)$th Bernoulli number of the rth order. These coefficients are useful in computing the derivatives of functions from tabulated values. In particular, they constitute a powerful tool for calculating values of functions for complex values of the argument when the function is known to be analytic in the neighborhood of the real axis and when the function has been tabulated for real arguments with sufficient accuracy and to a sufficiently small interval. Coefficients were computed with the aid of the recurrence formula $A_{n,r+1}=(nA_{n-1,r}-rA_{n,r})/(r+1)$ and further checked against the values obtained from the factorial polynomials. The usefulness of the table of coefficients here discussed will be enhanced by a table of $(x+iy)^n$ the preparation of which is now contemplated. (Received February 20, 1942.)

177. G. T. Miller and H. K. Hughes: *Analytic continuation of functions defined by factorial series.*

Given a function of z defined by a factorial series of the form: $\Omega(z) =\sum_{m=0}^{\infty}(-1)^{m(k+1)}g(m)/z\cdot(z+1)\cdots(z+m)$, where $g(w)$ is a single-valued analytic function of the complex variable $w=x+iy$, for $x\geqq -1/2$, and $g(w)$ is such that for all such x and for $|y|$ sufficiently large, $|g(x+iy)/g(x)|<M\exp(\pi(k+(1/2))-\epsilon)|y|$ then an analytic continuation of the function $\Omega(z)$ is given by the sum of two infinite integrals. This result may be considered to be an extension of a result obtained by H. K. Hughes (American Journal of Mathematics, vol. 53 (1931), pp. 771–775). The method of proof is based on the calculus of residues and bears a close analogy to the work of W. B. Ford (*Asymptotic Developments*, University of Michigan Studies, 1936) and to that of C. V. Newsom (American Journal of Mathematics, vol. 60 (1938), pp. 561–572). In case the function $g(w)$ has singularities, certain loop integrals are added to the expression already obtained. (Received March 10, 1942.)

178. Marston Morse and G. M. Ewing: *The non-regular case in the variational theory in large.*

The original treatment both of the absolute minimum theory and of the more general variational theory in the large made use of broken extremals. In the non-regular case these lost their most useful characteristics, making it necessary to treat the variational theory in the large in a new way. This has been done in three papers, the first on functional topology by Morse which makes the necessary alterations in the underlying topological theory, and in two papers by Morse and Ewing dealing with the local analytical aspects of the problem such as upper reducibility and the generalized Euler or homotopy theorem. The classical minimum theory including the Lindeberg theorem is first reviewed and refined, making a more systematic use of the hypothesis of convexity of the integrand than previously. In the second paper the more difficult question of upper-reducibility is taken up. The final theorems are on a general non-regular problem on a Riemannian manifold. (Received March 16, 1942.)

179. Isaac Opatowski: *Confluent hypergeometric functions and Markhoff chains.*

New simple relations between confluent hypergeometric functions are obtained by means of the Laplace transformation. The study is based on the function $[(n+m-1)!]^{-1}t^{n+m-1}e^{-t}\ {}_1F_1(m,\ n+m,\ t)\equiv\gamma_m(t,\ n)$, which has certain advantages of simplicity with respect to the Erdélyi's function $N_{k,m}$ (Mathematische Zeitschrift, vol. 42 (1936), pp. 125–143) for example, $\gamma_m(t,\ n)^*\gamma_p(t,\ q)=\gamma_{m+p}(t,\ n+g)$, where * means the convolution. Then γ_1 is the Pearson incomplete gamma function and $\gamma_{m+1}(t,\ n)$ is the m times repeated integral of $\gamma_1(t,\ n)$ between $t=0$ and $t=t$ (the integration being understood for an arbitrary m as the general derivative of Liouville-Riemann). The function $\gamma_m(t,\ n)$ is used to express the probability function $P_i(t)$

and calculate the moments of the Markoff chain (a generalization of the Poisson distribution): $dP_i/dt=k_iP_{i-1}-k_{i+1}P_i$, where $k_i=0$ and the positive constants k_i's have for $i=1, 2, 3, \cdots$ two different values only. (Received March 20, 1942.)

180. E. J. Pinney: *Calculus of variations in abstract spaces and related topics.* I.

The calculus of variations problem considered is that of minimizing the integral $I(y)=\int f(x, y(x), y_x(x))dV(x)$, where $y(x)$ is a function on an n-dimensional arithmetic space R to a linear topological space T which is required to contain a convex open set properly, $y_x(x)$ denotes the set of linear topological derivatives, $f(x, h, h_x)$ is a functional on $R, T, T, \cdots, T$, and the integral is an n-dimensional Lebesgue integral. The development necessitates the proof of certain theorems about the integral, and this first paper is devoted to that. The familar expression for the transformation of an integral under a mapping of the set on which it is defined to another (measurable) set is established. Green's theorem is established. A topological differential equivalent to a Fréchet differential when T is a Banach space is studied. The theorem of the mean is established for certain functionals, and continuity under the integral sign is considered. (Received March 14, 1942.)

181. E. J. Pinney: *Calculus of variations in abstract spaces and related topics.* II.

In this continuation of the first paper under this title, the theory there developed is applied to the calculus of variations problem mentioned there. The function $f(x, h, h_x)$ is defined, and an allowable class of functions $y(x)$ is defined. The abstract analogues of the necessary conditions of Euler, Weierstrass, Legendre, and Jacobi are established, and in addition the analogue of the necessary condition of Carathéodory is established. This condition, which reduces to that of Weierstrass in the case of the single integral, doesn't seem to be very well known as yet. Two "corner" conditions are established, and the condition of transversality on the boundaries is established. An investigation of sufficiency conditions is projected for the future. (Received March 14, 1942.)

182. Harry Pollard: *An inversion formula for the Stieltjes transform.*

This treatment of the Stieltjes transform (1) $f(x)=\int_0^\infty(x+t)^{-1}d\alpha(t)$ is intended to parallel a recent study of the Laplace integral made by Boas and Widder. The underlying idea, suggested by the work of Paley and Wiener, is to consider the iterate of the transform and to apply known results to this. In this way a new inversion operator can be derived for the transform (1). In terms of this operator necessary and sufficient conditions are obtained for the representation of a function as a Stieltjes transform with $\alpha(t)$ of general or of preassigned type. All these results depend on a knowledge of $f(x)$ on the real axis. (Received March 3, 1942.)

183. Harry Pollard: *On subseries of a convergent series.*

In a recent paper J. D. Hill (this Bulletin, vol. 48 (1942), p. 103) has defined the mean-value m of all the subseries of a given absolutely convergent series $\sum u_n$. In this case he has shown that $m=s/2$, where s is the sum of the series. By use of well known properties of the Rademacher functions this result is extended to the case of all convergent series for which $\sum|a_n|^2<\infty$. If $\sum|a_n|^2=\infty$, m fails to be defined, so that the extension is best possible. (Received March 4, 1942.)

184. George Pólya: *On the combinatory analysis of classifications and permutations.*

Let K_n denote the number of classifications of a set of n objects, $K_n=1, 2, 5, 15, \cdots$ for $n=1, 2, 3, 4, \cdots$ and let $K_0=1$. These K_n and connected combinatorial numbers occur in many questions, for example, K_n is the coefficient of $x^n e/n!$ in the development of exp (exp (x)). Let G be a permutation group of order g and the degree n, let $\mu_0=1$, and μ_k be the arithmetic mean of the kth powers of the traces of its g permutations. The μ_k are integers, $\mu_1 \geqq K_1$, $\mu_2 \geqq K_2, \cdots, \mu_n \geqq K_n$. If the equality is valid in all n inequalities, G is the symmetric group. Otherwise there is just one p, $0 \leqq p < n$, such that $\mu_p = K_p$, $\mu_{p+1} < K_{p+1}$ and G is p-fold transitive. (Received March 17, 1942.)

185. H. A. Rademacher: *On the Bloch-Landau constant.*

Landau in 1929 in a paper on the Bloch constant $\mathfrak{B}$ introduced another constant $\mathfrak{L}$ through the following definition: There exists a constant $\mathfrak{L}$ such that every function $w=f(z)$ which is regular in $|z|<1$ with $f'(0)=1$ covers completely, for any $\epsilon>0$, a circle of radius $\mathfrak{L}-\epsilon$ in the w-plane, whereas to every $\epsilon>0$ there exists a function of the same specifications whose values do not completely fill any circle of radius $\mathfrak{L}+\epsilon$. Landau gave the upper estimate $\mathfrak{L} \leqq 2^{2/3}\pi\Gamma^3(1/3)\Gamma^{-4}(1/4)=0.55488 \cdots$. In this paper we find $\mathfrak{L} \leqq \Gamma(5/6)\Gamma(1/3)\Gamma(1/6)^{-1}=0.54325 \cdots$. This upper bound is found by a suitable example obtained through the conformal mapping of a zero-angled circular triangle on a straight equilateral triangle. The calculations can be carried out with the help of a formula given by Ahlfors and Grunsky in 1937. (Received February 27, 1942.)

186. T. Radó: *On a problem of Geöcze.*

The Lebesgue area $A(S)$ of a continuous surface S is defined as the greatest lower bound of lim inf $E(\pi_n)$ for all sequences of polyhedra π_n converging to S in the Fréchet sense, where $E(\pi_n)$ stands for the elementary area of π_n. Denote by $A^*(S)$ the quantity obtained by requiring that the polyhedra π_n be inscribed in S. The problem of determining whether $A^*(S)=A(S)$ was first attacked by Geöcze (Mathematische und Naturwissenschaftlische Berichte aus Ungarn, vol. 26 (1910), pp. 1–88). The purpose of this paper is to extend the results of Geöcze by proving the following theorem. Let S be given by $z=f(x, y)$, where $f(x, y)$ is continuous in the unit square and is absolutely continuous in x for almost every y, or alternatively, absolutely continuous in y for almost every x. Then $A^*(S)=A(S)$. (Received February 28, 1942.)

187. P. V. Reichelderfer: *On bounded variation and absolute continuity for parametric representations of continuous surfaces.*

If S be a continuous surface in $x^1x^2x^3$-space given in parametric representation, denote by ${}^1T$, ${}^2T$, ${}^3T$ the corresponding representations for the projections of S upon the x^2x^3-, x^3x^1-, x^1x^2-planes, respectively. Definitions for essential variation and essential absolute continuity are given for ${}^1T$, ${}^2T$, ${}^3T$, based upon ideas developed by T. Radó and P. Reichelderfer (Transactions of this Society, vol. 49 (1941), pp. 258–307). An essential area $eA(S)$ is defined for S; it is a lower semi-continuous functional of S. A necessary and sufficient condition that $eA(S)$ be finite is that each of ${}^1T$, ${}^2T$, ${}^3T$ be of bounded essential variation. If $eA(S)$ is finite, then the generalized Jacobians

${}^1J$, ${}^2J$, ${}^3J$ for ${}^1T$, ${}^2T$, ${}^3T$ exist almost everywhere and are summable, and $eA(S) \geqq \iint [{}^1J^2 + {}^2J^2 + {}^3J^2]^{1/2}$. A necessary and sufficient condition that the sign of equality hold here is that each of ${}^1T$, ${}^2T$, ${}^3T$ be essentially absolutely continuous. If S has a non-parametric representation, then $eA(S)$ equals the Lebesgue area of S, and this theory becomes equivalent to that developed by Tonelli for this special case. (Received March 18, 1942.)

188. W. H. Roever: *Comment on the derivation of the law of perfect gases.*

In this paper the author shows that, to derive from the laws of Boyle and Charles the law of perfect gases, one encounters the problem of showing that the two two-parameter families of curves defined by these laws lie on a one-parameter family of surfaces. Also the derivation of the expression for the entropy of a perfect gas is discussed. (Received March 6, 1942.)

189. H. M. Schwartz: *On some general series expansions.* Preliminary report.

Using results obtained in two earlier papers (*Sequences of Stieltjes integrals*, this Bulletin, vol. 47 (1941), pp. 947–955, and abstract 48-1-55), the author studies some questions of convergence and summability of expansions in series of functions which form an orthogonal set with respect to a function of bounded variation, by methods similar to those employed by H. Lebesgue in his paper *Sur les intégrales singulières* (Annales de Toulouse, (3), vol. 1 (1909)). (Received March 20, 1942.)

190. M. F. Smiley: *A comparison of algebraic, metric, and lattice betweenness.*

It is shown that metric betweenness (see L. M. Blumenthal, *Distance Geometry*) and lattice betweenness (see Everett Pitcher and M. F. Smiley, *Transitivities of betweenness*, forthcoming in the Transactions of this Society) coincide in a complete normed real vector lattice if and only if this lattice is equivalent to an (L)-space (S. Kakutani, *Concrete representation of abstract* (L)*-spaces and the mean ergodic theorem*, Annals of Mathematics, (2), vol 42 (1941), pp. 523–537). The ranges of coincidence of algebraic and metric and of algebraic and lattice betweenness are also determined. Metric betweenness in the Banach space $C[0, 1]$ is investigated. (Received March 12, 1942.)

191. M. F. Smiley: *A remark on S. Kakutani's characterization of* (L)*-spaces.*

The important condition IX of Kakutani (*Concrete representation of abstract* (L)*-spaces and the mean ergodic theorem*, Annals of Mathematics, (2), vol. 42 (1941), pp. 523–537) may be replaced by the equivalent condition (1) $\|x-y\| = \|x \vee y - x \wedge y\|$. It is shown that this permits an economy of assumptions in §§3–5 of the paper cited. The proofs are based on the fact that the condition (1) is closely related to a fundamental property of modular functionals (G. Birkhoff, *Lattice Theory*, American Mathematical Society Colloquium Publications, vol. 25, 1940, p. 40). (Received March 12, 1942.)

192. W. S. Snyder: *Non-parametric surfaces of bounded variation.*

The paper shows that if $f(x, y)$ is of bounded variation in the sense of Hardy and Krause (Hobson, *Functions of a Real Variable*, vol. 1, 3rd edition, 1927, §254), then the Lebesgue area of the surface $z=f(x, y)$ is equal to the Burkill integral of a certain rectangle function analogous to the rectangle function of de Geöcze (Saks, *Theory of the Integral*, p. 171). The method shows that the area of the surface may be calculated by a single passage to the limit from the areas of certain polyhedra inscribed in the surface. (Received March 20, 1942.)

193. Otto Szász: *On sequences of polynomials and the distribution of their zeros.*

The author considers sequences of polynomials $P_n(z)=c_{n0}+c_{n1}z+c_{n2}z^2+\cdots$ with increasing degrees $m_1<m_2<\cdots$, respectively, where the roots of P_n lie in a half-plane containing the origin on its boundary line, the direction of which may depend on n. If the first three coefficients are uniformly bounded, and the c_{n0} are bounded away from 0, then the sequence is uniformly bounded in any finite domain. If, in addition lim $c_{n\nu}$ exists as $n\rightarrow\infty$ for each ν, then the sequence converges uniformly in any finite domain, thus representing an entire function, the convergence exponent and order of which are at most two. In particular if the sequence is a subsequence of partial sums of a formal power series, then this series represents an entire function of the described type. Some previous results of G. Pólya are thus generalized. (Received March 6, 1942.)

194. S. M. Ulam and D. H. Hyers: *Approximate isometries of the space of continuous functions.*

A transformation of a metric space E into itself will be called an ϵ-isometry if it changes distances by an amount less than ϵ (see abstract 47-9-427). Properties of such transformations are studied in the case where E is the space of continuous functions. (Received March 24, 1942.)

195. F. A. Valentine: *On the extension of a vector function so as to preserve a Lipschitz condition.*

Let $f(x)$ be a vector function mapping a set S in the euclidean plane into a set S' in the euclidean plane. Moreover $f(x)$ satisfies a Lipschitz condition of the form $|f(x_1)-f(x_2)|\leqq K|x_1-x_2|$ for all pairs x_1 and x_2 in S, where $|x-y|$ is the euclidean distance between the points x and y. If T is any set containing S, then $f(x)$ can be extended to T so as to preserve the Lipschitz condition. The extension $[f(x), x\in T]$ can be defined so as to be contained in any prescribed bounded, closed, convex set containing $S'\equiv[f(x), x\in S]$. This extension is a consequence of the following: *Consider a set of circles in the plane, such that there is a point in common to all of the circles. Move these circles to new positions in the plane, subject to the condition that the distance between any pair of centers is not increased. Then all of the circles in the new positions will still have a point in common.* After proving this result for three circles, one can prove it for an arbitrary set by using a theorem of Helly for families of bounded, closed, convex sets. (Received March 6, 1942.)

196. J. L. Walsh: *On the overconvergence, degree of convergence, and zeros of sequences of analytic functions.*

The entire theory of overconvergence as developed by Porter, Jentzsch, Ostrowski, Bourion, and others is here generalized and unified by the concept of exact harmonic majorant of a sequence of analytic functions. If the function $V(z)$ is harmonic in a region R of the z-plane, if the functions $F_n(z)$ are locally single-valued and analytic in R except for branch points, and if $|F_n(z)|$ is single-valued in R, then $V(z)$ is said to be an exact harmonic majorant of the sequence $F_n(z)$ in R provided one has $\limsup_{n\to\infty}[\max |F_n(z)|, z \text{ on } Q] = [\max e^{V(z)}, z \text{ on } Q]$ for every continuum Q (not a single point) in R. Applications of this concept involve degree of convergence and properties of the zeros of functions, and include maximal sequences of polynomials and of other rational functions, and many other sequences of analytic functions. (Received March 16, 1942.)

197. M. S. Webster: *A convergence theorem for certain Lagrange interpolation polynomials.*

A convergence theorem for a sequence of Lagrange interpolation polynomials based on the zeros of a sequence of certain Jacobi polynomials is proved. The method and result are similar to a theorem of Grünwald (this Bulletin, vol. 47, (1941), pp. 271–275). (Received March 19, 1942.)

198. Hermann Weyl: *Solution of the simplest boundary-layer problems in hydrodynamics.*

For some simple configurations the hydrodynamic boundary-layer problem can be reduced to a non-linear ordinary differential equation of third order involving a parameter λ. For $\lambda=0$ and $1/2$, solution may be obtained by a rapidly converging process of alternating successive approximations. The general case is attacked by a suitable adaptation of the method of fixed points of transformations in functional spaces. (Received February 28, 1942.)

199. František Wolf: *On the limits of harmonic and analytic functions along radii which form a set of positive measure.*

If $u(r, \theta) = \log|f(re^{i\theta})|$ and $f(z)$ is analytic in the unit circle $r<1$, $u(r, \theta) \leqq M/(1-r)^n$ for any M and n, and $\limsup_{r\to 1} u(r, \theta) \leqq 0$ for $\theta \subset E$, $|E|>0$, then $\limsup u(r, \theta) \leqq 0$ in any sector at almost all points of E. Hence if $u(r, \theta)$ is harmonic and satisfies the conditions of the theorem, then $u(r, \theta)$ and its conjugate $v(r, \theta)$ have finite limits in any sector at almost all points of E. This follows from above by the well known results of Privaloff (Recueil Mathématique de Moscou, vol. 91 (1923), p. 232) and Fatou. Another corollary is: If $f(z)$ is analytic in $|z|<1$, $|f(z)| \leqq \exp [M/(1-r)^n]$, and $\lim_{r\to 1} f(re^{i\theta}) = 0$ for $\theta \subset E$, $|E|>0$, then $f(z) \equiv 0$. (Received March 20, 1942.)

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200. Stefan Bergman: *Determination of pressure in the two-dimensional flow of an incompressible perfect fluid.*

The author considers a flow of an incompressible perfect fluid around a wing profile. The pressure distribution is determined by the function $W(z)$ which maps the exterior $\mathfrak{E}$ of the wing profile onto the exterior $\mathfrak{R}$ of a circle. Of particular interest is the evaluation of $W(z)$ on the boundary in the neighborhood of the vertex O. Let boundary in the neighborhood of O be formed by two circular arcs CO and BO which make an angle α at O. Let O and D be the intersections of the circles on which the arcs CO and BO lie. Suppose that arcs CD and BD lie inside of the profile (Hypothesis