

this book is also the source of its weakness, namely, that it is in last analysis only an introductory outline. It is written with the traditional French clarity, and will be a useful addition to the library of the mathematician taking an interest in modern physics.

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Topological Groups. By L. Pontrjagin. Translated by Emma Lehmer. Princeton, University Press, 1939. 9+299 pp.

The topological group is a combination of two fundamental mathematical concepts—group and topological space. A topological group G is a group and at the same time a topological space in which the group operations in G are continuous. Historically, the concept arose from the study of groups of continuous transformations. Pontrjagin gives an axiomatic treatment of topological groups. Later he points out their connections with continuous transformations as well as with other older concepts. In the language of the author: "This book is intended for the reader with rather modest mathematical preparation." This is accomplished very successfully by both the included material and its organization. All material needed is precisely formulated, and in most cases proofs are given. The understanding of the text is enhanced by the inclusion of seventy-five examples, which deal largely with real numbers, matrices, and vector spaces. The author points out questions left unanswered in most of the general problems discussed.

The first three chapters give an excellent introduction to topological groups. Chapter I discusses the usual topics in elementary abstract group theory. These include normal subgroups, factor groups, homomorphisms, the center of a group, direct products, and commutative groups. In Chapter II a topological space is defined by means of axioms in terms of the closure of a set. An equivalent neighborhood definition is set up and is used extensively. Among the concepts studied are connectedness, regularity, second axiom of countability, compactness, and topological products. Continuous mappings are introduced early and have a prominent place in the chapter. Chapter III contains the first steps in the theory of topological groups. The fundamental relations holding for abstract groups and topological spaces are adapted to topological groups. Additional concepts introduced include interior (open) mappings and local isomorphisms.

After the introductory material in the first three chapters, the reader may proceed to any one of Chapters IV, VI, or VIII. Chapter IV proves that any compact group satisfying the second axiom of

countability admits a complete system of representations. The establishment of an invariant measure on the group is the first step in the proof. Haar gave the first construction of such a measure, but the author gives von Neumann's construction of invariant integration because it is simpler. The proof also contains an exposition of the results of Peter and Weyl concerning the completeness of the system of functions arising from irreducible representations.

The fundamental results of Chapter V, which depend on the theory of representations of Peter and Weyl, are due to the author. Locally compact commutative groups satisfying the second axiom of countability are investigated principally through the construction of a character group. It is shown that to such a group G there corresponds a group X of the same kind, which is called the character group of G . Since the correspondence between G and X is symmetric, any question concerning one of the groups reduces to the corresponding question about the other. When G is compact its character group X is shown to be discrete, and conversely. Hence the study of a compact G reduces to the study of an abstract group.

Some of the fundamental problems of Lie groups (which may be discrete) are solved in Chapter VI. The results of the chapter are intended primarily as preparatory material for Chapter VII, in which it is shown that the study of compact groups satisfying the second axiom of countability can be reduced to the study of Lie groups and their limits. The last named chapter contains a positive solution of Hilbert's fifth problem (is every parameter group a Lie group?) for compact groups. A positive solution was given in Chapter V for locally compact commutative groups.

Chapter VIII develops the following results (organized and formulated by Schreier) concerning the connection in the large between locally isomorphic groups: Let Δ be the aggregate of all connected, locally connected, and locally simply connected groups which are locally isomorphic with one such group G . There exists one and only one (up to an isomorphism) simply connected group G^* in Δ , which is called the universal covering group of G . Moreover, any group G' of Δ can be written in the form G^*/N , where N is a discrete central normal subgroup isomorphic with the fundamental group of G' . The chapter contains a satisfying treatment of the fundamental group and universal covering space of a topological space.

The main object of Chapter IX is to show that the local study of Lie groups can be reduced to the study of infinitesimal groups. While the deeper results of Killing, Cartan, and Weyl in the theory of infinitesimal groups are not taken up here, some are stated without

proof. A complete classification of semi-simple Lie groups is given in this way. Among the other topics discussed are compact Lie groups and groups of continuous transformations.

This book should prove of invaluable aid both to the beginner in the field of topological groups and to the more advanced student. While a number of typographical errors were found, they should not prove confusing to an alert reader.

W. T. PUCKETT, JR.

Statistical Methods. By Paul R. Rider. New York, Wiley, 1939. 9+220 pp.

This book has two aims: first, to serve as a textbook for an elementary course in statistics, and second, to help students with some previous knowledge of statistics to gain an insight into the more modern methods. It proceeds from some preliminary development of the classical theory, through such topics as "Student's" distribution, to the various significance tests associated with the χ^2 and Fisher z distributions. The notation of the calculus is used in a number of the formulas. However, so much emphasis is placed on the practical applications of the theory that the statistical worker who uses the book as a laboratory manual will probably not find the mathematical notation disconcerting, no matter what his previous mathematical training may have been.

The classical theory is presented in Chapters I-IV and the first part of Chapter V. Chapters I and II are concerned with the elementary theory of frequency distributions, and with averages and moments. Chapter III contains a discussion of regression, with an exposition of Fisher's method of handling the normal equations in the case of multiple regression. Chapter IV is on simple and multiple correlation, and deals entirely with the observational theory. In Chapter V we find brief descriptive treatments of such topics as the continuous approximation to the binomial distribution, the normal and Gram-Charlier type A distributions, the significance of the difference between two means and two proportions, the significance of correlation coefficients (tested by means of Fisher's logarithmic transformation); and there is also an introduction to the theory of confidence limits. In Chapter VI, we are introduced to "Student's" t distribution which is then applied to appropriate problems, such as testing the significance of regression coefficients. At the beginning of Chapter VII the χ^2 distribution is described, and there follow the usual applications to homogeneity tests, tests of goodness of fit, and contingency