For the analytic discussion of the space P, rectangular Cartesian coordinates x, y, z are used (E is $z=0$) and homogeneous coordinates p_0, p_1, p_2, p_3, where $x=p_2/p_1$, $y=p_3/p_1$, $z=p_0/p_1$. The exclusion of the line at infinity in E from P permits the normalizing condition $p_0^2+p_1^2=1$, so that we may write $p_0=\cos\lambda$, $p_1=\sin\lambda$; then λ, p_2, p_3 define a point in P or, equivalently, a displacement in E.

An algebra of displacements is obtained by means of biquaternions of the form $p=p_0e_0+p_1e_1+\epsilon(p_2e_2+p_3e_3)$, where p_0, p_1, p_2, p_3 are real or complex numbers (the homogeneous coordinates as above) and $e_0, e_1, e_2, e_3, \epsilon$ satisfy $e_0=1, e_1^2=e_2^2=e_3^2=-1$, $e_2e_3=-e_3e_2=e_1, \cdots, \epsilon e_1=e_1\epsilon, \cdots, \epsilon^2=0$. It is evident that the product of two biquaternions of the above form is a biquaternion of the same form. The conjugate $\bar{p}$ of p is obtained by changing the signs of the coefficients of e_1, e_2, e_3.

Now if a displacement p is applied to a rigid body, a point initially at l is displaced to r, where $r=\bar{p}lp$. If this displacement is followed by a second displacement q, then the final position of the point is t, where $t=\bar{p}^*lp^*$, in which $p^*=pq$; in fact, the resultant of two displacements is their product.

The "geometry" of the space P consists of those properties of figures in P which remain invariant under certain transformations. These transformations are as follows. Let us apply in order a displacement b_l, a displacement p, and a displacement b_r. The resultant is a displacement $p^*=b_lpb_r$. This is the required transformation of P into itself: these transformations form a group G_6, since each of b_l, b_r depends on three parameters. These transformations leave invariant the points $(0, 0, i, 1)$, $(0, 0, 1, i)$ and the planes $x_0 \pm ix_1=0$, where x_0, x_1, x_2, x_3 are homogeneous coordinates. The resultant geometry the author calls *quasielliptic*, on account of a limiting connection with elliptic geometry.

The preceding remarks give some idea of the first half of the book (Algebraischer Teil). The second half of the book deals with the differential geometry of the quasielliptic space P. A curve is given by writing the homogeneous coordinates p_0, p_1, p_2, p_3 as functions of a parameter λ. The privileged basic parameter is that λ which appears in connection with the normalizing condition $p_0^2+p_1^2=1$, so that the equations of the curve are $p_0=\cos\lambda$, $p_1=\sin\lambda$, $p_2=p_2(\lambda)$, $p_3=p_3(\lambda)$. Quasicurvature and quasitorsion are defined. Surfaces are also considered and curves on surfaces.

The book is based on lectures delivered in 1938 at the University of Hamburg. One may smile at the encouraging prefatorial remark: "An Vorkenntnissen ist nur ein wenig analytische Geometrie und Infinitesimalrechnung erwünscht." For the book is not easy reading, seeming to lack purpose, so that the reader may well ask himself from time to time: "What is the goal of all this?" However, there is much meat in the little volume, as we might expect from the reputation of its author in geometry.

In addition to the regular exposition, there are a number of sections headed "Aufgaben und Lehrsätze."

J. L. SYNGE

Probability, Statistics, and Truth. By Richard von Mises. Translated by J. Neyman, D. Scholl, and E. Rabinowitsch. New York, Macmillan, 1939. 16+323 pp.

This translation follows closely the original, *Wahrscheinlichkeit, Statistik und Wahrheit,* 2d edition, 1936, which I reviewed in the Journal of the American Statistical Association (vol. 31 (1936), pp. 758–759). However, in the preface the author states, "I have added several paragraphs in the English edition (pp. 141–147). These deal with certain investigations of A. H. Copeland, E. Tornier, and A. Wald, which were published after the appearance of the second German edition."

The purpose of Mises is to present in language as non-technical as possible the fundamental principles of probability with applications to statistics and mechanics. In my opinion, this has been done with marked success. A reader with some background in science, philosophy, or logic should be able to read with understanding the greater part of the book, even though his available mathematics is extremely limited.

Instead of basing a theory of probability upon "equally likely cases," Mises has proposed the use of "*collectives*," sequences of numbers, such as 4, 6, 1, $\cdots$, which might be obtained from throwing a die—these sequences required to satisfy two requirements: Axiom I. If, among the first n numbers, a specified number appears m times, then as n increases indefinitely m/n approaches a limit. Axiom II. "*Randomness.*" If from such a sequence, a subsequence is formed by any "place selection" which picks out a term without knowledge of its value, then the foregoing limits are unchanged. This might be looked upon as an axiomatic approach in which Mises *considers only those sequences of numbers or vectors which conform to the above two axioms.* In the "Third Lecture," Mises discusses in detail objections that have been urged against his theory, some authors declaring collectives non-existent. Mises states (p. 143): "Copeland and, later, Wald have proved the correctness of the following proposition for which Copeland found the proof for the case of two attributes only, and which Wald generalized: Given an arbitrary enumerable set of place selections, it is possible to define a collective (even a non-enumerable set of such collectives), in which the relative frequencies of particular attributes tend to the limits prescribed by the given distribution and this is not affected by any of the place selections in the given set. I have interpreted my randomness axiom by saying that the limiting values of the relative frequencies should remain insensitive to all place selections occurring in the particular problem under consideration. It is clear that in any special problem the number of place selections is either finite or, at the most, enumerable. It is even possible to postulate that the system of *all* definable selections is at most enumerable. This postulate agrees well with the modern formal construction of logic. . . . A certain restriction of these propositions is necessary only if the set of attributes considered is *infinite*, ... On the whole, the very thorough and ingenious investigations of Copeland and Wald can be said to dispel completely all the mathematical objections against the use of the randomness axiom which we have previously mentioned." Among the 103 numbered references in the appendix—some referring to more than one author—are several which discuss the foundations of the Mises theory. Another very recent publication is a set of papers by P. Cantelli, W. Feller, M. Fréchet, R. de Mises, J. F. Steffensen, and A. Wald on *Les Fondements du Calcul des Probabilités*, Actualités Scientifiques et Industrielles, no. 735, papers written for the Geneva Conference on the Théorie des Probabilités, 1937.

Mises distinguishes his theory from that based upon "equally likely cases," from theories involving Axiom I, without incorporating Axiom II, and from theories admitting Axiom II in very restricted form. He also takes the position that *probability is not a branch of the theory of additive set functions.* "It remains in all circumstances a theory of certain observable phenomena, which are idealized in the concept of a collective." Mises finds a distinction here that some others have failed to find. At all events, his *collective must first be set up.* After that is done, other branches of mathematics may be used to solve a proposed problem. According to Mises, *no probability exists* unless it is possible to set up a *collective* to describe it.

This review, written for the Bulletin has stressed theoretical considerations. But in Mises' book, the gentle reader will find a delightful non-technical introduction to probability, with carefully selected simple illustrations and a minimum of formulas and mathematical display. He will be led to the heart of many of the most interesting and important problems in probability itself, and then on to statistics in general and to the statistical problems of theoretical physics or mechanics. Some section headings in the latter topics are the following: Linked events, Lexis theory and the laws of large numbers, Mendel's theory of heredity, Industrial statistics, Galton's board, The second law of thermodynamics, Machines dependent on chance, Small causes and large consequences, Kinetic theory of gases, Brownian motion, Entropy theory and Markoff chains, Radio-active radiations, Quantum theory, The renunciation of causality, Heisenberg's uncertainty principle.

EDWARD L. DODD

La Notion de Point Irrégulier dans le Problème de Dirichlet. By Florin Vasilesco. (Actualités Scientifiques et Industrielles, no. 660.) Paris, Hermann, 1938. 61 pp.

This is the twelfth of the booklets containing "Exposés sur la Théorie des Fonctions" published under the direction of Paul Montel as a part of the series named above. This number contains an interesting compilation of the results of some recent researches on the Dirichlet problem. The discussion makes frequent use of material treated in another booklet by the same author: *La Notion de Capacité* (Actualités Scientifiques et Industrielles, no. 571, 1937).

The first chapter is devoted to a brief discussion of artificial (that is, removable) singularities of harmonic functions of three variables. The author takes as his starting point a theorem to the effect that a function continuous at a point P and bounded and harmonic elsewhere in the neighborhood of P is harmonic at P. As this theorem is attributed to Picard (1923) the author is evidently unfamiliar with an earlier paper by Bôcher in which the same result is established (this Bulletin, vol. 9 (1903), p. 455 ff.; the priority of Bôcher's theorem has already been noted by Raynor, ibid., vol. 32 (1926), p. 537 ff. and by Kellogg, ibid., vol. 32 (1926), p. 664 ff.).

The second chapter, which is the longest of the pamphlet, is devoted to the study of the solution of the generalized Dirichlet problem. The author discusses such topics as conditions for regularity and irregularity of boundary points (especially those expressed in terms of capacity or the conductor potential) barriers, the generalized Green's function, Lebesgue's example of an irregular point, and Kellogg's lemma and some of its corollaries.

Chapter III is devoted mainly to a discussion of the results concerning balayage published by Frostman in 1935.

Chapter IV contains a brief summary of the contents of a booklet by de la Vallée Poussin, *Les Nouvelles Méthodes de la Théorie du Potentiel et le Problème de Dirichlet Généralisé* (Actualités Scientifiques et Industrielles, no. 578, Paris, 1937). One of the topics discussed is the lightening of the requirement of continuity at a multiple boundary point of a spatial domain. Apparently both de la Vallée Poussin and Vasilesco have overlooked an earlier discussion of the Dirichlet problem for three dimensional domains with multiple boundary points (Perkins, Transactions of this Society, vol. 38 (1935), p. 106 ff.).

The remaining chapters of the booklet contain brief sketches of some of the most recent work on topics connected with the Dirichlet problem. Much of this exposition is based on researches published in 1938. Chapter V is concerned principally with Marcel Riesz's notion of generalized potentials of order α (an extension of earlier work by Frostman); Chapter VI is devoted primarily to an account of recent work on balayage by Brelot. Chapter VII is unique in that it makes use of logarithmic capacity; it contains some results (due mainly to Frostman) concerning functions of a complex variable.