Kopf's trisection of the angle is improved by B. J. Topel in such a way that the maximum error is reduced from 15″ to 0.5″.

In the concluding paper, Professor Menger advances a number of interesting suggestions on the logic of doubting, commanding, and wishing. In his logic of the doubtful, a proposition must be contained in one of three mutually exclusive classes, according as it is asserted, doubted, or negated. This logic differs from those ordinarily proposed in that the class of a compound proposition is not uniquely determined by the classes of its constituents; two doubtful propositions may be related to each other in a number of ways, according to the classes in which various of their compounds belong. To introduce option into logic, preference is formalized by using a fixed proposition A (for example, "I shall be glad") and regarding p as desirable when p implies A; it is urged that only doubtful propositions are objects of wishes and commands. Perhaps the most suggestive remark made in connection with optative logic —of which several simple cases are treated—is that the relation "M is preferable to (that is, more valuable than) N" furnishes a partial ordering of economic goods in which ordinal incomparability is transitive.

Especially in the first and third papers, there are numerous misprints; except occasionally in the third, none were noticed which seemed likely to confuse an attentive reader.

F. A. FICKEN

Ebene Kinematik. By W. Blaschke. (Hamburger mathematische Einzelschriften, no. 25.) Leipzig, Teubner, 1938. 56 pp.

"Plane Kinematics" is a subject usually regarded as a part of mechanics. It starts with the simple concept of the plane displacement of a rigid body, and leads on through consideration of velocity and acceleration into technical complications of importance to engineers. In the book under review no such treatment is attempted, although the title might lead one to expect it. On the contrary, dependence on time is not included, so that we have to consider only the simplest kinematical concept, namely, the displacement of a rigid body in a plane. The superstructure based on that concept is essentially not that of the engineer (a scant three pages are devoted to linkages) but of the modern geometer in search of a manifold.

Any displacement of a rigid body in a plane may be specified by three parameters (for example, the components of displacement of any point in the body and the angle of rotation). Hence these displacements constitute a manifold or space of three dimensions. It is with the geometry of this three-space that the book is concerned. We must indicate in some detail how the author realizes this three-space.

Let E be the plane in which the displacements take place, and let S be an euclidean three-space containing E. First consider displacements in E other than pure translations. Any such displacement is a rotation about some point A of E through some angle 2ω. Let us take the point B in S on the normal to E at A, at a distance $\cot \omega$ below E. Then B represents the displacement in the sense that B is uniquely defined by, and uniquely defines, the displacement. Actually, the author gives a very simple geometrical construction by which the displacement may be found when B is given, but we shall not describe it here.

Thus the totality of displacements in E (omitting translations) corresponds to the whole of the euclidean space S. Translations correspond to points at infinity of S (with the exception of points at infinity in E), and so the complete representative space for all displacements is a projective three-space P, from which, however, the line at infinity in E is omitted.

For the analytic discussion of the space P, rectangular Cartesian coordinates x, y, z are used (E is $z=0$) and homogeneous coordinates p_0, p_1, p_2, p_3, where $x=p_2/p_1$, $y=p_3/p_1$, $z=p_0/p_1$. The exclusion of the line at infinity in E from P permits the normalizing condition $p_0^2+p_1^2=1$, so that we may write $p_0=\cos\lambda$, $p_1=\sin\lambda$; then λ, p_2, p_3 define a point in P or, equivalently, a displacement in E.

An algebra of displacements is obtained by means of biquaternions of the form $p=p_0e_0+p_1e_1+\epsilon(p_2e_2+p_3e_3)$, where p_0, p_1, p_2, p_3 are real or complex numbers (the homogeneous coordinates as above) and $e_0, e_1, e_2, e_3, \epsilon$ satisfy $e_0=1, e_1^2=e_2^2=e_3^2=-1$, $e_2e_3=-e_3e_2=e_1, \cdots, \epsilon e_1=e_1\epsilon, \cdots, \epsilon^2=0$. It is evident that the product of two biquaternions of the above form is a biquaternion of the same form. The conjugate $\bar{p}$ of p is obtained by changing the signs of the coefficients of e_1, e_2, e_3.

Now if a displacement p is applied to a rigid body, a point initially at l is displaced to r, where $r=\bar{p}lp$. If this displacement is followed by a second displacement q, then the final position of the point is t, where $t=\bar{p}^*lp^*$, in which $p^*=pq$; in fact, the resultant of two displacements is their product.

The "geometry" of the space P consists of those properties of figures in P which remain invariant under certain transformations. These transformations are as follows. Let us apply in order a displacement b_l, a displacement p, and a displacement b_r. The resultant is a displacement $p^*=b_lpb_r$. This is the required transformation of P into itself: these transformations form a group G_6, since each of b_l, b_r depends on three parameters. These transformations leave invariant the points $(0, 0, i, 1)$, $(0, 0, 1, i)$ and the planes $x_0\pm ix_1=0$, where x_0, x_1, x_2, x_3 are homogeneous coordinates. The resultant geometry the author calls *quasielliptic*, on account of a limiting connection with elliptic geometry.

The preceding remarks give some idea of the first half of the book (Algebraischer Teil). The second half of the book deals with the differential geometry of the quasielliptic space P. A curve is given by writing the homogeneous coordinates p_0, p_1, p_2, p_3 as functions of a parameter λ. The privileged basic parameter is that λ which appears in connection with the normalizing condition $p_0^2+p_1^2=1$, so that the equations of the curve are $p_0=\cos\lambda$, $p_1=\sin\lambda$, $p_2=p_2(\lambda)$, $p_3=p_3(\lambda)$. Quasicurvature and quasitorsion are defined. Surfaces are also considered and curves on surfaces.

The book is based on lectures delivered in 1938 at the University of Hamburg. One may smile at the encouraging prefatorial remark: "An Vorkenntnissen ist nur ein wenig analytische Geometrie und Infinitesimalrechnung erwünscht." For the book is not easy reading, seeming to lack purpose, so that the reader may well ask himself from time to time: "What is the goal of all this?" However, there is much meat in the little volume, as we might expect from the reputation of its author in geometry.

In addition to the regular exposition, there are a number of sections headed "Aufgaben und Lehrsätze."

J. L. SYNGE

Probability, Statistics, and Truth. By Richard von Mises. Translated by J. Neyman, D. Scholl, and E. Rabinowitsch. New York, Macmillan, 1939. 16+323 pp.

This translation follows closely the original, *Wahrscheinlichkeit, Statistik und Wahrheit*, 2d edition, 1936, which I reviewed in the Journal of the American Statistical Association (vol. 31 (1936), pp. 758–759). However, in the preface the author states, "I have added several paragraphs in the English edition (pp. 141–147). These deal with certain investigations of A. H. Copeland, E. Tornier, and A. Wald, which were published after the appearance of the second German edition."

The purpose of Mises is to present in language as non-technical as possible the