The proof is too complicated to be sketched here. However it is worth saying that what Gentzen does is to describe a means of attaching an ordinal number (less than $\epsilon_0$) to any proof of number theory. He then describes how, if one had a proof of a contradiction, one could find a second proof of a contradiction having a smaller ordinal number than the first proof.

J. BARKLEY ROSSER

*Reports of a Mathematical Colloquium.* Series 2, no. 1. Edited by Karl Menger. Notre Dame University Press, 1939. 64 pp.

This booklet of seven papers begins a continuation of the earlier series of reports issued from 1928 to 1936 by the Vienna Colloquium under the leadership of Professor Menger.

"Stability of Limited Competition and Cooperation," by G. C. Evans and Kenneth May, deals with two producers. Under simplifying assumptions, conditions are found on the coefficients of the demand and cost functions in order that an equilibrium point be possible, that this point be competitive or cooperative, and that the equilibrium be stable. Under strong hypotheses, similar methods are applied to labor, leading to the conclusion that a union able to control the labor supply for a given industry can make the introduction of machinery unprofitable to the entrepreneur.

"On Linear Sets in Metric Spaces," by Karl Menger and Arthur Milgram, contains four theorems, of which a special consequence is the known theorem that, in a complete and convex metric space, any two distinct points are joined by a subset congruent to a segment of the euclidean line.

The third paper is "Partially Ordered Sets, Separating Systems and Inductiveness," by A. N. Milgram. It is unfortunate that this interesting and substantial paper gives the impression of having been written hurriedly and printed without proofreading. The results given seem to be new and significant.

Essentially, certain portions of the Dedekind theory of the continuum are so adjusted as to be useful in studying partially ordered sets. Let $A$ be a partially ordered set. A subset $L$ is called a lower section of $A$ if $a \varepsilon L$ and $b<a$ imply $b \varepsilon L$; if $B$ is a subset of $A$, the set of those elements $a$ of $A$ with the property that $a \leqq b$ for every $b$ in $B$ is called the under section of $B$. In terms of these concepts, it is found possible to define well-ordered subsets and to associate with each element of $A$ a unique subset which is well-ordered and has other useful properties; this association is produced once with and once without an application of transfinite induction. Separation—analogous to that effected in the continuum by the rational numbers—is defined intrinsically and extrinsically, the equivalence of the definitions is proved, and the powers of well-ordered subsets are compared with the powers of systems of separating sets. If $A$ has a denumerable separating system, it is shown that $A$ can be mapped on an interval of the continuum in such a way that order-relations are preserved. Several applications are given, chiefly to problems in topology.

"Postulates for the Ratio of Division," by B. J. Topel, deals with a set of elements and a real-valued function $f$ of trios of these elements. $f$ is subject to postulates which arise in a natural way from the properties of the ratio in which a point divides a segment in elementary geometry. It is shown that the set can be so metrized that $f$ is the quotient of two distances. Limiting processes, orientation, and $f$-preserving transformations are studied, and other sets of postulates are considered.

Frederick P. Jenks proposes a set of postulates for Bolyai-Lobachevsky geometry based on the operations of joining and intersecting, and shows that these postulates are sufficient for the usual discussion of betweenness.

Kopf's trisection of the angle is improved by B. J. Topel in such a way that the maximum error is reduced from 15″ to 0.5″.

In the concluding paper, Professor Menger advances a number of interesting suggestions on the logic of doubting, commanding, and wishing. In his logic of the doubtful, a proposition must be contained in one of three mutually exclusive classes, according as it is asserted, doubted, or negated. This logic differs from those ordinarily proposed in that the class of a compound proposition is not uniquely determined by the classes of its constituents; two doubtful propositions may be related to each other in a number of ways, according to the classes in which various of their compounds belong. To introduce option into logic, preference is formalized by using a fixed proposition $A$ (for example, "I shall be glad") and regarding $p$ as desirable when $p$ implies $A$; it is urged that only doubtful propositions are objects of wishes and commands. Perhaps the most suggestive remark made in connection with optative logic —of which several simple cases are treated—is that the relation "$M$ is preferable to (that is, more valuable than) $N$" furnishes a partial ordering of economic goods in which ordinal incomparability is transitive.

Especially in the first and third papers, there are numerous misprints; except occasionally in the third, none were noticed which seemed likely to confuse an attentive reader.

F. A. FICKEN

*Ebene Kinematik.* By W. Blaschke. (Hamburger mathematische Einzelschriften, no. 25.) Leipzig, Teubner, 1938. 56 pp.

"Plane Kinematics" is a subject usually regarded as a part of mechanics. It starts with the simple concept of the plane displacement of a rigid body, and leads on through consideration of velocity and acceleration into technical complications of importance to engineers. In the book under review no such treatment is attempted, although the title might lead one to expect it. On the contrary, dependence on time is not included, so that we have to consider only the simplest kinematical concept, namely, the displacement of a rigid body in a plane. The superstructure based on that concept is essentially not that of the engineer (a scant three pages are devoted to linkages) but of the modern geometer in search of a manifold.

Any displacement of a rigid body in a plane may be specified by three parameters (for example, the components of displacement of any point in the body and the angle of rotation). Hence these displacements constitute a manifold or space of three dimensions. It is with the geometry of this three-space that the book is concerned. We must indicate in some detail how the author realizes this three-space.

Let $E$ be the plane in which the displacements take place, and let $S$ be an euclidean three-space containing $E$. First consider displacements in $E$ other than pure translations. Any such displacement is a rotation about some point $A$ of $E$ through some angle $2\omega$. Let us take the point $B$ in $S$ on the normal to $E$ at $A$, at a distance $\cot \omega$ below $E$. Then $B$ represents the displacement in the sense that $B$ is uniquely defined by, and uniquely defines, the displacement. Actually, the author gives a very simple geometrical construction by which the displacement may be found when $B$ is given, but we shall not describe it here.

Thus the totality of displacements in $E$ (omitting translations) corresponds to the whole of the euclidean space $S$. Translations correspond to points at infinity of $S$ (with the exception of points at infinity in $E$), and so the complete representative space for all displacements is a projective three-space $P$, from which, however, the line at infinity in $E$ is omitted.