

principle in the dynamics of the atmosphere. It is also gratifying to find at last in meteorological literature the name of Kelvin mentioned (p. 58) in connection with the problem of convective instability of moist air, which he treated for the first time in 1865. In his treatment of the heat balance of the atmosphere the author fails to stress a fundamental difficulty, which is our ignorance, at present, of the radiative behaviour of water vapor under atmospheric conditions. The result of Hergessel referred to on page 76, that in a semi-grey atmosphere at radiative equilibrium there would exist a uniform temperature of -54°C , has been shown to be incorrect.*

On the whole the book offers a convenient exposition of present-day problems of dynamic meteorology.

C. L. PEKERIS

Technique de la Méthode des Moindres Carrés. By Henri Mineur. (Monographies des Probabilités, publiées sous la direction de M. Émile Borel, no. 2.) Paris, Gauthier-Villars, 1938. 8+93 pp.

Mineur states in the preface of his book that it should be possible for the reader to learn to use the method of least squares without understanding the theory behind it. In fact, the theory of the various operations is not fully explained until the fifth chapter. For this reason the mathematician will perhaps find it more satisfactory to proceed directly from the first chapter to the fifth and then read the second, third, and fourth. We shall follow this order in discussing the topics which Mineur treats.

The first chapter consists of an example for which it is desired to fit a linear equation to a set of data. The attempt to make such an equation conform exactly leads in general to an incompatible system. The equation which is the best fit in the sense of least squares is shown in Chapter 5 to result from the so-called normal equations which are compatible and linear. The resolution of a linear system by means of determinants is cumbersome and hence the author presents the Gaussian method which constitutes the principal part of the technique. The method of least squares is shown to be equivalent to finding the equation which produces the minimum probable error. Also in Chapter 5 one finds a discussion of such concepts as mean, standard deviation, probable error of a single measurement, and probable error of the mean of a set of measurements. In fact the author gives a brief but clear exposition of the elements of statistics. It is, however, surprising that simple, multiple, and partial correlation are omitted.

Chapter 2 contains a detailed description of the method of tabulating the data, forming the normal equations, and solving them. Mineur fails to note that much of this tabulation is unnecessary when a computing machine is used. For example, by means of a machine it is possible to obtain a sum of products as a single operation without recording the individual products. In Chapter 3 the author discusses the nature of errors of measurement. He also indicates how the method of least squares can be applied to nonlinear equations. In Chapter 4 he applies his method to the solution of a problem in stellar statistics.

On the whole, this is a readable and useful book.

A. H. COPELAND

British Association for the Advancement of Science: Mathematical Tables. Volume 6: *Bessel Functions.* Part 1: *Functions of Orders Zero and Unity.* Cambridge, University Press; New York, Macmillan, 1937. 20+288 pp.

The preface to this volume opens with the words: "It is with the satisfaction of

* C. L. Pekeris, *Gerlands Beiträge*, vol. 28 (1930), p. 377.

keeping a long-anticipated engagement that a Committee of the British Association issues its first volume of tables of Bessel functions. Half a century ago, the Committee decided that the tabulation of Bessel functions was the most useful undertaking that it could promote." In fact, one of the tables in the *Treatise of Bessel Functions*, by Gray and Matthews, is credited to the Reports of the British Association for 1889, and in the revised edition of 1922 the Committee is quoted as intending "to publish at an early date a volume of fairly complete tables of Bessel functions."

But with the years the undertaking has grown until we now have one volume published, a second, on functions of other integral orders, in "an advanced state of preparation," and a third taking shape.

One hundred and seventy pages are devoted to the functions $J_0(x)$ and $J_1(x)$, which are tabulated to ten places of decimals (with second differences) at intervals of 0.001 for x from $x=0$ to $x=16$ and at intervals of 0.01 for x from $x=16$ to $x=25$. Then follow tables for the zeros of $J_0(x)$ and $J_1(x)$, the values of the functions $Y_0(x)$, $Y_1(x)$, $I_0(x)$, $I_1(x)$, $K_0(x)$, $K_1(x)$, together with auxiliary functions and coefficients useful in interpolation. The definitions of these functions through differential equations, power series, and recurrence formulas are conveniently and concisely given.

Mechanically the tables should be easy to use. The entries are in blocks of five with the integral part of each entry given only at the head of the block, unless a change in "characteristic" requires more frequent entry. In the opinion of the reviewer, this is an ideal arrangement for tables of this sort.

The comparisons made with other tables and the listing of errors found in earlier tables are both interesting and valuable.

The Committee has had the courage to say, "There is thus every reason to believe that the tables are completely free from error." Let us hope that nothing more serious has crept in than the trivial missing decimal point in the value of $J_1(7.840)$ which the eye casually picks up on page 80.

The British Association is making a valuable contribution in preparing these authoritative source books of numerical tables. The future volumes in the series on Bessel functions will be awaited with interest.

G. R. CLEMENTS

Essai sur les Fondements de la Géométrie Euclidienne. By Julien Malengreau. Lausanne, Payot, 1938. 311 pp.

This book is intended as an introduction to a more complete treatise on geometry. The author gives a set of twenty-nine postulates, involving the undefined terms *point* and *distance*, and develops the resulting theory.

The problems of the consistency and independence of the postulates are not considered. As a matter of fact it is fairly obvious that rational euclidean 3-dimensional geometry R_3 (that is, the subset of a cartesian 3-space consisting of those points all of whose coordinates are rational) is an example of a space in which postulates 1 to 28 are satisfied. Postulate 29 (il existe au moins un pentapoint parfait) requires that the space be at least 4-dimensional, and 1-29 hold true in R_4 .

There is no axiom of *continuity* (or *completeness*) such as the Dedekind cut axiom. On the other hand there is no closure axiom limiting the set of points on a line to a countable set. Likewise, there is no closure axiom limiting the dimension.

The treatment is elementary, as the proofs are all based on rational arithmetic. There are fifty-six figures which illustrate many of the postulates and theorems.

J. H. ROBERTS