ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross-references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

1. A. T. Craig: *On correlation due to common elements.*

In this paper linear functions of independently observed values of a chance variable x are considered when these functions have in common certain of the observed values. A necessary and sufficient condition that the functions shall have a linear regression system is derived. An investigation is also made of the regression system of real symmetric quadratic forms of normally and independently distributed variables. (Received November 19, 1937.)

2. H. L. Rietz: *On the correlation of a mean and standard deviation in small samples drawn from a certain non-normal population.*

The parent distributions with which this paper is concerned are given by urn schemata devised by the author some years ago to give meaning to measures of correlation in relation to certain given probabilities. The theoretical distributions resulting from the urn schemata were published in the Annals of Mathematics, vol. 21 (1920), pp. 306–322. In 1925, Leone E. Chesire prepared a master's thesis relating to small samples drawn from these non-normal distributions. Certain data that were obtained by the experimental sampling of Miss Chesire seem to be appropriate material for the study of the correlation of mean and standard deviation, and the corresponding regressions. The main object of the present paper is to report certain results of such a study. (Received November 5, 1937.)

3. F. D. Rigby: *Note on the axioms for Boolean algebras.*

Let $E=[0, 1]$ and let $f(x, y)$ be any one of the sixteen functions in EE to E. If the Sheffer stroke operation x/y is defined by $f(x, y)$, then the axioms of Boolean algebras hold in only two cases. A systematic study of the axioms which fail to hold is made with reference to their independence and interrelations. While the axioms of Sheffer can be proved completely independent by using algebras of two elements, this is not the case for the axioms of Huntington's fourth set. These can be shown to be completely independent by aid of algebras of three or more distinct elements. (Received November 10, 1937.)

4. C. E. Sealander: *A particular third order irregular boundary value problem.* Preliminary report.

The differential system $u'''+p(x)u'+[\rho^3+q(x)]u=0$, $u(0)=u'(0)=u(\pi)=0$, is considered, $q(x)$ being a function with power series expansion in powers of x^3 only, and $p(x)$ being x times such a function. If $u_n(x)$ denotes the solutions of the system, and if the series $\sum a_n u_n(x)$ converges uniformly for real values of x in the interval $(0, a)$, $a < \pi$, then the series converges uniformly, for complex values of x, within an equilateral triangle with center at the origin and one vertex on the axis of reals. It is shown that the series converges uniformly to a function $f(x) = x^2\phi(x^3)$, ϕ having a power series expansion in powers of x^3 only. Finally it is shown that the formal series expansion of $f(x)$ converges uniformly to $f(x)$ everywhere in the interval $(0, a)$ when suitable requirements of analyticity and continuity are imposed on the functions involved. (Received November 1, 1937.)

5. Gerhard Tintner: *A dynamic theory of duopoly.*

Duopoly exists if there are only two sellers in the market. Following the work of G. C. Evans (*Mathematical Introduction to Economics*, pp. 22 ff.) and C. F. Roos (*A dynamical theory of economics*, Journal of Political Economy, vol. 35 (1927), pp. 632 ff.; *A mathematical theory of competition*, American Journal of Mathematics, vol. 47 (1925), pp. 163 ff.; *Generalized Lagrange problems in the calculus of variations*, Transactions of this Society, vol. 30 (1928), pp. 360 ff.) the problem can be stated in a somewhat more general form: Let one duopolist maximize his profit in the interval $(0, n)$, $P = \int_0^n [p(t)x(t) - C(t)]dt$, where p is the market price, x his production, C his costs. The following relations exist: $x(t) + g[x] = f[p]$, where g is the production of his rival and a functional of his own production $x(s)$; and f is the demand, a functional of the price $p(s)$. Further, $C(t) = h[x]$, where h is a functional of $x(s)$, $0 \geqq s \geqq t$. The necessary conditions for a maximum are $x(\alpha) = \int_\alpha^n l(\beta) f'[p;\alpha]d\beta$ and $p(\alpha) + l(\alpha) = \int_\alpha^n \{h'[x;\alpha] - l(\beta)g'[x;\alpha]\}d\beta$. Here l is a Lagrange multiplier (see H. Hahn, *Ueber die Lagrangesche Multiplikatorenmethode*, Sitzungsberichte Akademie der Wissenschaften, Wien, Abstrakt IIa, vol. 131 (1922), pp. 531 ff.). Here f' is the functional derivative at the point α, as are h' and g', while $p(\gamma)$ and $x(\gamma)$ are defined for $0 \geqq \gamma \geqq \beta$; finally, $0 \geqq \alpha \geqq n$. (Received October 30, 1937.)

6. A. A. Albert: *Quadratic null forms over a function field.*

Let L be a finite field, y an indeterminate over L, $F = L(y)$, K be algebraic of finite degree over F. Quadratic forms over K are considered and it is shown that every form in five or more variables is a null form. The proof uses the splitting field theory of E. Witt (Mathematische Annalen, vol. 110 (1934)), except in the case where L has characteristic two. Here the theorem above is proved valid for L any perfect field. In fact let L be perfect of characteristic p, $q = p^e$. Then it is shown that all forms $a_1x_1{}^q + \cdots + a_mx_m{}^q$ over K are null forms if $m > q$, but there exist non-null forms of this type for every $m \leqq q$. This result applied for $q = p = 2$ is seen to give the main theorem. (Received November 11, 1937.)

7. E. T. Bell: *Polynomial approximations for elliptic functions.*

The method of a previous paper (Transactions of this Society, vol. 36 (1934), pp. 841–852) is applied to obtain successive approximations, with respect to powers of the modulus, of the elliptic functions. (Received November 5, 1937.)

8. G. A. Bliss, M. F. Smiley, and Leonidas Alaoglu: *Fields for multiple integrals of the calculus of variations.*

Two types of fields for multiple integrals of the calculus of variations have been described in recent papers by Carathéodory and Weyl. In the present paper the authors exhibit the most general type of multiple integrals which are independent of the path, and show that the fields associated with them are more general than those of

Carathéodory and Weyl. Special attention is given to the case when there are two independent variables and two dependent functions, a case which has been previously discussed in quite different fashion by Hadamard and Hilbert. (Received November 22, 1937.)

9. J. W. Calkin: *Second order partial differential operators of elliptic type. Closure.*

Let E be an open set in $(x_1, \cdots, x_n)$-space. Let p and q be real-valued Lebesgue-measurable functions defined on E, $p>0$, q and p_{kk}, $(k=1, 2, \cdots, n)$, bounded on every closed set interior to E. Let $\mathfrak{D}$ be the subset of the complex Hilbert space $\mathfrak{L}_2(E)$, each of whose elements f contains a function f^0 with the following properties: on almost all parallels to the x_k-axis which intersect E, f^0 and $f_k{}^0$ are absolutely continuous on every closed interval interior to E, $(k=1, 2, \cdots, n)$; on every bounded closed set S interior to E the Lebesgue integrals $\int_S |f_k{}^0|^2 dS$, $\int_S |f_{kk}{}^0|^2 dS$ exist, $(k=1, 2, \cdots, n)$; the Lebesgue integral $\int_E |L(f^0)|^2 dE$ exists, where $L(f^0) = -\sum_{k=1}^n (pf_k^0)_k + qf^0$. Let T be the transformation with domain $\mathfrak{D}$ which takes f into the element of $\mathfrak{L}_2(E)$ containing $L(f^0)$. Then $\mathfrak{D}$ is dense in $\mathfrak{L}_2(E)$, $T \equiv \tilde{T}$, $T \supseteqq T^*$. The condition which characterizes those elements g of $\mathfrak{D}$ which belong to the domain of T^* is a restriction on the behavior of g^0 in the "neighborhood" of the boundary of E. These results are fundamental for the author's contemplated extension of the theory described in abstracts 43-3-114, 43-3-209, 43-9-323, to the case of n variables, $n \geqq 3$. (Received November 24, 1937.)

10. B. H. Camp: *Note on the probability distribution of a product.*

Two transformations are presented which are specially useful for this sort of distribution, the one producing a distribution theorem for a product corresponding to any given distribution theorem for a sum, and vice versa, the other producing the volume of a differential element in N dimensions by means of a transformation from a simpler differential volume. Some useful probability distributions of the product, which contain some known distributions as special cases, are thus derived. (Received November 1, 1937.)

11. R. V. Churchill: *Concerning Laplace transforms of periodic functions.*

Let $F(t)$ be a bounded integrable periodic function with period a. Its Laplace transform $f(z)$, together with its analytic continuation, is analytic in every finite region except possibly at the points $2n\pi i/a$, $(n=0, \pm 1, \pm 2, \cdots)$, which are simple poles with residues C_n, the Fourier constants of $F(t)$, if $C_n \neq 0$, and regular points if $C_n=0$; C_{-n} is the conjugate of C_n; also $|x| \cdot |f(x+iy)|$ is bounded in the half-planes $x \geqq b$ and $x \leqq -b$ for every $b>0$. Let $F(t)$ and its first derivative be piecewise continuous. Then $f(z)$ satisfies the additional conditions that its complex inversion integral converges for $t>0$, and that $|y_n| \cdot |f(x+iy_n)|$, where $y_n=(2n+1)\pi i/a$, is bounded for all x and n. Moreover, all of the above conditions which pertain to $f(z)$ alone form a set sufficient to insure the periodicity with period a and boundedness of the inverse transform $F(t)$, for $t>0$. Both the form and validity of the Fourier series representation of $F(t)$ under either of the above sets of conditions, on $F(t)$ or its transform, follow as a special case of a known series inversion of the Laplace transformation. (Received November 23, 1937.)

12. J. M. Clarkson: *A line transformation related to reciprocal polars of a quadric surface.*

Through an arbitrary line t of space pass two planes tangent to a fixed quadric Q at points R, S, respectively. From a fixed point O not on Q the points R, S are projected into points R', S', respectively, on Q. The tangent planes to Q at $R'S'$ meet in a line t', which is the transform of t by the line transformation. The singular lines and the invariant lines of the transformation are discussed. (Received November 18, 1937.)

13. A. H. Copeland: *A general existence theorem for simultaneous equations.*

Sufficient conditions for the simultaneous vanishing of n functions of n variables are developed in this paper. It is assumed that the functions possess first partial derivatives and that the Jacobian is non-vanishing throughout a given region R. Let g denote half the sum of the squares of the n functions. It is also assumed that a number c exists such that the region for which $g<c$ is non-vacuous and has no boundary points in common with the boundary of R. These conditions are sufficient, but probably not necessary, to insure the simultaneous vanishing of the functions. As an example it is shown that all the conditions are satisfied by a system of linear equations for which the determinant of the coefficients is not zero. As a second example an extremely simple proof of the implicit function theorem is obtained. In fact this entire paper is shorter and simpler than the treatment of the implicit function theorem alone, as it usually appears in texts on the advanced calculus. (Received November 23, 1937.)

14. H. B. Curry: *On the reduction of Gentzen's calculus LJ.*

In 1934 (Mathematische Zeitschrift, vol. 39, pp. 176–210, 405–431) Gentzen set up a "calculus *LJ*" and proved its equivalence to a form of the Heyting calculus which he called the "calculus *LHJ*." This note gives an alternative reduction of *LJ* to *LHJ*. The advantages of the new method are these: (1) it makes use of known general theorems about the Heyting calculus; (2) it shows that the derivation of any scheme in *LJ* can be made on the basis of only those axioms of *LHJ* which concern implication, together with those which actually occur in the scheme being derived. This, in combination with Gentzen's "Hauptsatz," gives a new proof of the fact that a similar separation applies to all the theorems of the Heyting calculus. (Received November 20, 1937.)

15. H. T. Davis and H. E. Jones: *Some results in the theory of random series.*

By a random series $\{y(s)\}$ is meant a series of N items, such that the autocorrelation function is zero for all lags, that is $\sum_{s=1}^{N} y(s)y(s+t)=0,\ t\neq 0$. This paper studies the properties of linear combinations of the items of random series by means of the lag-correlation functions between the series and their differences and between the differences of different orders. The results obtained are extended to continuous variables through the solution of a difference equation due to Yule. Applications to the theory of economic time series and to other statistical data are indicated. (Received November 23, 1937.)

16. J. J. De Cicco: *The inverse of Meusnier's theorem in the geometry of element series.*

This is a continuation of the paper, *The differential geometry of element series,* which was presented to this Society in March, 1937. The problem is to find the most general system of series with the Meusnier property. (For the analogous problem in ordinary space, see *The inverse of Meusnier's theorem* by Edward Kasner, this Bulletin, vol. 14 (1908), pp. 461–465). This deals with the curvature of the series of the system which have a common element and a common tangent turbine, and may be stated in three equivalent ways: 1. The reciprocal of the curvature varies as the sine of half the angle between the osculating flat field and a fixed flat field. 2. The osculating limaçon series generate a spherical field. 3. The centers of the central turbines of the osculating limaçon series form a straight line. The most general system of series with the Meusnier property is $Av''+Bw''+C=0$ where A, B, C are functions of u, v, w, v', w'. In connection with this system, there is derived a theory of *most-turbinal series* which includes, as a special case, the ∞^2 geodesic series of a field. (Received November 22, 1937.)

17. W. L. Duren: *Contractible problems of Bolza.* II.

Contractible problems of Bolza as described in a previous paper (abstract 42-11-418) did not include problems of isoperimetric type. An extension of the method described there has been found to lead to an analog of the fundamental sufficiency theorem in contracted fields of a more general type which are suitable for isoperimetric problems. (Received November 23, 1937.)

18. Churchill Eisenhart: *The power function of the χ^2-test.*

If we have N' values of a random variate classified into n' cells for which the probabilities are p_r^0, $(r=1, 2, \cdots, n')$, on the null hypothesis, H_0, and if we reject H_0 whenever $\chi^2 \geqq \chi_0^2$, then $P\{\chi^2 \geqq \chi_0^2 | H_0\}$ gives the probability of rejecting H_0 falsely with the χ^2-test. When N' is large this probability is obtainable (approximately) from the limiting distribution of χ^2 found by Karl Pearson. If, on the other hand, an alternative hypothesis, $H \neq H_0$, specifying the probabilities p_r, $(p_r \neq p_r^0$ for all $r)$, is true, then $P\{\chi^2 \geqq \chi_0^2 | H\}$ for this same $\chi_1^2 = \chi^2(H_0)$ gives the probability of rejecting H_0 when H is true, that is, the probability of detecting H with the χ^2-test. By using an artifice the writer has found that, for N' sufficiently large, $p(\chi^2 | H)d\chi^2$ is given (approximately) by R. A. Fisher's B^2-distribution with the substitutions $B^2 = \chi^2$ and $\beta^2 = \sum_r \alpha_r^2 / p_r^0$, where the α_r are defined by $p_r = p_r^0 + \alpha_r/(N')^{1/2}$, from which $P\{\chi^2 \geqq \chi_0^2 | H\}$ is obtained by integrating from χ_0^2 to infinity, the integral considered as a function of χ_0^2 being the power function of the test. (Received November 18, 1937.)

19. W. W. Flexner: *Complexes whose vertices have manifolds as linked complexes.* Preliminary report.

Let an n-sphere be a generalized n-manifold of order null and define a generalized n-manifold of order p as a connected simplicial n-complex such that the linked complex of each of its vertices is a generalized $(n-1)$-manifold of order always less than p and, once at least, equal to $p-1$. It is shown that the join of two such manifolds is a generalized manifold whose order is given in terms of orders and dimensions of its components. The regular subdivision, K', of a generalized manifold K of order p is then a generalized manifold such that for $q > p-2$ the linked complexes of the vertices of K' on q-simplexes of K are spheres. So the generalized manifold divides into two complexes, one a relative n-manifold (Lefschetz), the other composed of the neighborhoods of simplexes whose linked complexes are not spheres. For order 2, the complex which these have in common is the sum, S, of those linked complexes which are not spheres. The p and $n-p$ Betti numbers of the orientable generalized manifold of order 2 are connected by a relation involving also the p Betti number of S and the seam-homology numbers (Alexandroff-Hopf, p. 289) of S. (Received November 23, 1937.)

20. M. M. Flood: *Strict equivalence of matric pencils.* Preliminary report.

A rational canonical form for an arbitrary matric pencil is obtained without a preliminary transformation of indeterminates. This leads to a simultaneous classification of singular and non-singular matric pencils. The conjunctive equivalence of hermitian pencils and the strict equivalence of matric nets are also considered. (Received November 26, 1937.)

21. Lois W. Griffiths: *On hypergroups, multigroups, and product systems.*

A product system is a system of classes which is closed with respect to an associative and commutative addition process, and which is closed with respect to an associative multiplication process which is distributive with respect to the addition process. A division system is a product system for which either division, or the existence of elements analogous to the identity and inverse elements of group theory, is postulated. A division system is a generalization of the abstract system group, and of hypergroup as defined by Marty (Comptes Rendus, vol. 201 (1935), pp. 636–638). It is proved that the abstract systems, multigroup as defined by Ore (Duke Mathematical Journal, vol. 3 (1937), pp. 149–174), and hypergroup as defined by Wall (American Journal of Mathematics, vol. 59 (1937), pp. 77–98), are types of Marty hypergroups. There is defined an algebra associated with a product system, and, in particular, a hypergroup algebra. Properties of elements of the hypergroup algebra give, in particular, properties of the elements of the hypergroup. (Received November 20, 1937.)

22. D. C. Harkin: *On secant and tangent coefficients.*

Taylor-Maclaurin expansions of elliptic functions for unit modulus involve as coefficients the secant (Euler) and tangent coefficients. The theory of elliptic functions thus suggests a rapid method for the computation of these numbers. The results of Scherk (correcting Euler) are thus checked and extended. (Received November 24, 1937.)

23. Olaf Helmer: *The syntax of a language with infinite expressions.*

All languages which have so far been made the subject of syntactical investigation have had one feature in common; their expressions consisted of a finite number of symbols. In this paper the syntax of a language with expressions of infinite length is dealt with. Various possibilities as to the kind of infinite expressions that might be introduced suggest themselves, for example, infinite logical sums and products, or predicates with infinitely many argument places. The author builds up a language, which is modeled on the example of Carnap's languages I and II but which admits of numerical expressions of infinite length, namely, of infinite dual fractions. Like language I, this language contains a K-operator, and it does not go beyond the elementary functional calculus. The requirement of definiteness, on the other hand, is given up altogether. After enumerating the symbols and laying down the formation rules, a set of axioms and a number of rules of deduction are given. The next step is the arithmetization of the syntax of this language. Since this can be carried out within the elementary theory of real numbers, of which the language itself is a representation, one can formulate the syntax of the language in itself. The remainder of the paper is concerned with the question as to the extent to which Gödel's results can be transferred to this language. (Received November 20, 1937.)

24. Paul Henle: *On the categoricity of the postulates for Boolean algebra.*

This paper constitutes a proof that, if to any one of the usual sets of postulates for Boolean algebra a postulate be added specifying the number of elements in the system as some definite finite number, then the resulting set of postulates is categorical. The proof begins by exhausting all possibilities in the case of an algebra of two elements, and then proceeds by induction on powers of two. Extensions of the theorem to algebras of an infinite number of elements are suggested. (Received November 23, 1937.)

25. T. R. Hollcroft: *The maximum number of contacts of two algebraic surfaces.*

Given two algebraic surfaces M and N, of respective orders μ and ν, whose intersection curve consists of α components of orders n_i and genera p_i, the maximum number K of contacts of M and N is proved to be $\mu\nu(\mu+\nu-4)/2+\alpha$. When $\alpha>1$, M and N have a contact at each of the Q intersections of the components and a number P of additional possible contacts. Here Q and P are both functions of n_i and p_i as well as of μ and ν, but $K=Q+P$ is a function of μ, ν, and α, only; that is, the maximum number of contacts is independent of the orders and genera of the components. Expressions for the maximum number of contacts are obtained also when common points A_i of M and N are of multiplicities r_i on M and s_i on N. Two cases are treated, A_i on distinct components and A_i on pairs of components. Numerical applications are made to quadrics and other surfaces. The maximum number of contacts can be attained by surfaces of low order. The maximum number of stationary contacts of M and N depends on the solution of the problem of finding the maximum number of cusps of a plane curve of given order and genus. This problem has not been completely solved. The maximum number of stationary contacts of M and N, however, decreases as α decreases. (Received November 26, 1937.)

26. Harold Hotelling and L. R. Frankel: *The transformation of statistics to simplify their distributions.*

When a statistic has a non-normal distribution that approaches normality for large samples, there are advantages in the use of a function of the statistic having a more nearly or exactly normal distribution. These advantages consist partly in the decreased use of tables required in tests of significance, and partly in the possibilities of further statistical studies based on a combination with other similar functions, as when one fits a regression equation to a number of correlation coefficients pertaining to various time intervals. A statistic calculated from statistics as if they were direct observations is, as a rule, best interpreted if the statistics of which it is a function have been normalized in the manner indicated. Similarly, it is advantageous to transform a statistic whose ultimate distribution is that of χ to one having exactly, or more nearly, this limiting distribution. These transformations are illustrated in this paper by the asymptotic expansion of a normalized variate in terms of the Student t, and by that of a variate having the χ distribution in terms of the generalized Student statistic T introduced in 1931 as a substitute for "coefficients of racial likeness." From tables it appears that the first few terms of the expansions provide approximations satisfactory for many purposes. (Received November 15, 1937.)

27. Edward Kasner and J. J. De Cicco: *The classification of element transformations by means of isogonal and equi-tangential series.*

This paper is a continuation of the paper, *The geometry of isogonal and equi-tangential series*, by Kasner, Transactions of this Society, vol. 42 (1937), pp. 94–106. In this paper, the authors give a classification of all the element transformations with respect to the precise number of unions or isogonal series (equi-tangential series) which are converted into isogonal series (equi-tangential series). The main results are: 1. All the element transformations are classified into four mutually exclusive non-null classes with respect to the number of isogonal series which become isogonal series. The classes 1^0, 2^0, 3^0, 4^0 are those which transform, respectively, all ∞^∞, $2\infty^2$, ∞^3, ∞^4 isogonal series into isogonal series. 2. All the element transformations are classified into five mutually exclusive non-null classes with respect to the number of equi-tangential series which become equi-tangential series. The classes 1^0, 2^0, 3^0, 4^0, 5^0 are those which transform, respectively, all ∞^∞, ∞^2, $2\infty^2$, ∞^3, ∞^4 equi-tangential series into equi-tangential series. (Received November 22, 1937.)

28. Edward Kasner and J. J. De Cicco: *The geometry of the whirl-motion group G_6. Elementary invariants.*

This paper is a continuation of the paper, *The group of turns and slides and the geometry of turbines* by Kasner, American Journal of Mathematics, vol. 33 (1911), pp. 193–202. A turn T_α turns each element of the plane about its point through a constant angle α. A slide S_k slides each element along its line a constant distance k. These transformations generate a three parameter group which is called the whirl group. The whirl group and the rigid motion group together generate a six parameter group G_6 which is termed the whirl-motion group. It is the purpose of the authors to study the geometry of G_6. A turbine is the ∞^1 elements which are obtained by applying a turn T_α to the elements of a circle. A flat field consists of the ∞^2 elements which are co-circular with a fixed element. Under G_6, turbines and flat fields are converted into turbines and flat fields respectively. The elementary invariants between elements, turbines, and flat fields are obtained. (Received November 22, 1937.)

29. O. E. Lancaster: *Non-linear algebraic difference equations with formal solutions of the same type as the formal solutions of linear homogeneous difference equations.*

The investigations of this paper treat the following problems: Given a non-linear algebraic difference equation (1) $F(x, y(x), y(x+1), \cdots, y(x+n))=0$, where F is a polynomial with rational coefficients in its arguments x, $y(x)$, $y(x+1)$, $\cdots$, $y(x+m)$, what is the nature of F when the difference equation has solutions in common with a linear homogeneous algebraic difference equation? What is the nature of F when

(1) has a formal solution of the same general type as that of the formal solutions of linear homogeneous algebraic difference equations? And, how many linearly independent solutions of this type may a given equation possess? The study of the formal problem yields the following results: First, there is a large class of homogeneous difference equations such that any equation of the class has q and only q linearly independent solutions of the desired type, where q is the product of the degree and the order of the equation. Second, there are homogeneous difference equations which do not possess any solutions of the required type. Third, there are homogeneous equations which have an infinite number of linearly independent solutions of the given type. (Received November 20, 1937.)

30. C. H. Langford: *A theorem on deducibility for second-order functions.*

Take any set of postulates for a dense series which is complete with respect to first-order functions (see Lewis and Langford, *Symbolic Logic*, chapter 12) and add to this set the condition of Dedekind section. Take any matrix on the base of the set $F(\phi_1, \phi_2, \cdots, x_1, x_2, \cdots)$, where the ϕ's are functions of one variable on each of which the hypothesis of Dedekind's condition is imposed (that is, the ϕ's determine lower segments, and jointly line segments). Consider the class of all functions that can be derived by generalizing the variables in such a matrix. Each of these functions has its truth-value determined by the set in question, and the set may therefore be said to be complete with respect to statements about a finite number of line segments. In the presence of Dedekind's condition, the ϕ's can be progressively eliminated by a Skolem reduction procedure. (Received November 20, 1937.)

31. Jakob Levitzki: *On the equivalence of the nilpotent elements of a semi-simple ring.*

In the present note it is shown that each class of equivalent nilpotent elements of a semi-simple ring can be characterized by certain characteristic numbers; two nilpotent elements belong to the same class if and only if their characteristic numbers coincide. This is achieved by reducing each nilpotent element to a certain normal form. As a consequence one finds that the number of classes is finite. By applying the results to square matrices in a commutative or non-commutative field it is found that each nilpotent matrix can be transformed into a Jordan form. (Received November 6, 1937.)

32. R. G. Lubben: *Concerning upper semicontinuous collections and the decomposition of points of normal spaces.*

If S is a semicompletely normal Hausdorff space, there exist "portions" of it that may be decomposed into smaller portions and reassembled into larger portions. Among these are the points of the space, which in general are decomposable, and certain ideal elements, called "boundary points" (see abstract 43-9-344, this Bulletin). Every portion of S is decomposable into points which are "atomic" relative to S. Every collection of portions of S may be regarded as the set of all points of a space H Fréchet in which the operation of derivation of point sets is defined in terms of that operation for S. An "amalgamation point of S" is essentially "a perfectly compact portion of S." The theory of "upper semicontinuous" collections of amalgamation points gives a powerful tool for the study of upper semicontinuous collections of point sets; gives criteria for the embedding of S in perfectly compact Hausdorff spaces, and for extending upper semicontinuous collections of point sets in S to upper semicontinuous collections in such spaces; and further, it gives methods for determining inverse decomposition spaces. The space of all atomic points of S is a perfectly compact Hausdorff space, and may be regarded as a universal inverse decomposition space for S. (Received November 22, 1937.)

33. N. H. McCoy and Deane Montgomery: *Subrings of direct sums and related topics.*

Let there be given a set of rings S_i, $(i \subset I)$, where I is an arbitrary range of indices. By the *direct sum* of the rings S_i, $(i \subset I)$, is meant the ring of all functions defined on I such that on i the functional values are in S_i. One of the leading results of the present paper is the theorem: *Every commutative ring without nilpotent elements is isomorphic to a subring of a direct sum of fields.* This is a generalization of results obtained by Stone (Transactions of this Society, vol. 40 (1936), pp. 37–111), by Köthe (Jahresbericht der Deutsche Mathematiker-Vereinigung, vol. 47 (1937), pp. 125–144) and by the present authors (Duke Mathematical Journal, vol. 3 (1937), pp. 455–459). The authors also show that an arbitrary ring is isomorphic to a subring of a direct sum of irreducible rings, and other related results. In particular, they establish some theorems concerning ideals in various types of rings. Finally, some of these results are applied to furnish characterizations of the commutative regular rings as defined by J. von Neumann (Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 707–713). (Received November 3, 1937.)

34. W. H. McEwen: *On the simultaneous approximation of a function and its derivatives by sums of Birkhoff type.*

Let $\{u_i(x)\}$ be the sequence of characteristic solutions of a given nth order linear differential system $L(u)+\lambda u=0$, $W_j(u)=0$, $(j=1, 2, \cdots, n)$, with regular boundary conditions defined on (a, b). By a sum of Birkhoff type of the order N is meant simply a linear combination $S_N(x)$ of $u_1, \cdots, u_N$. In this paper hypotheses are obtained under which a given function $f(x)$ and its first n derivatives can be uniformly approximated on $a \leqq x \leqq b$ in the form $|f^{(k)}-S_N^{(k)}| \leqq \epsilon_N \rightarrow 0$, $(k=0, 1, \cdots, n)$. (Received November 23, 1937.)

35. Saunders MacLane: *The uniqueness of the power series representation of certain fields and valuations.*

Krull has introduced generalized valuations, which are functions V on a field K to an ordered abelian group Γ, such that $V(a+b) \geqq \min (Va, Vb)$ and $V(ab)=(Va)+(Vb)$. The valuation V is of rank one and discrete if Γ consists of all lexicographically ordered linear forms $a_1\alpha_1+ \cdots +a_m\alpha_m$, with integral coefficients a_i. Then V can be decomposed into m rank 1 valuations $V^{(i)}$, corresponding to a sequence of residue-class fields, $K, K', \cdots, K^{(m-1)}, K^{(m)}$. It is known that such a field K has a smallest extension $\bar{K}$ which is perfect with respect to V; that is, which is topologically complete with respect to each component rank 1 valuation $V^{(i)}$. Is this smallest perfect extension $\bar{K}$ uniquely determined, up to analytic isomorphism, by K? The present paper proves $\bar{K}$ unique if $K^{(m-1)}$ has characteristic 0, and shows by examples that $\bar{K}$ is not generally unique when $K^{(m-1)}$ has characteristic p. The construction of these examples depends on a new method for explicitly constructing the perfect extension $\bar{K}$ of a given K. This method is also used to obtain some structure theorems for p-adic fields, which are related to the known representation of certain perfect fields as power series fields. (Received November 22, 1937.)

36. H. M. MacNeille: *The completion of a Boolean ring and its application to integration.*

Consider a Boolean ring, K, of elements $a, b, c, \cdots$, in which an absolute value is defined such that: $|a| \geqq 0$, $|a| = 0$ if, and only if, $a = 0$ and $|a| = |a - ab| + |ab|$. Fundamental sequences are defined in K. All the postulated properties of K are extended to L, the set of fundamental sequences in K. Then K is imbedded in L and L is closed to further extension by this method. If K is a Boolean ring of point sets and K_λ the set of limit elements of sequences in K which converge in the point set sense, then (abstract 43-3-132) K_λ is a Boolean ring containing K as a subring. In general, K_λ is not closed to further extension by this method and the iterates of this extension generate the smallest Borel system containing K. If sequences in K that are null in the point set sense are null fundamental sequences, then the point sets obtained by iterating the λ-process can be identified with the abstract elements of L. No element of L is omitted. This identification permits the establishment of theorems about integration and measure in the abstract set L. The Boolean ring of half-open elementary figures with rational end points in euclidean n-space, volume being the absolute value, satisfies all the assumptions on K. (Received November 23, 1937.)

37. Karl Menger: *A theorem on relations and its applications to covering theorems of topology.*

The author considers three given abstract sets A, B, and $R \subseteqq A \times B$, and two given sets $\mathfrak{A}^*$ and $\mathfrak{B}^*$ whose elements are subsets of A and B, respectively, and which satisfy the following three conditions. I. If $B^* \epsilon \mathfrak{B}^*$ and with each $b \epsilon B^*$ there is associated a set $A_b \epsilon \mathfrak{A}^*$, then $\sum A_b \epsilon \mathfrak{A}^*$, where the sum is taken over all $b \epsilon \mathfrak{B}^*$. II. If $A^* \epsilon \mathfrak{A}^*$ and there exists a 1-to-1 correspondence between the sets A^* and B^* such that pairs of corresponding elements belong to R, then $B^* \epsilon \mathfrak{B}^*$. III. The vacuous set belongs to $\mathfrak{B}^*$. If $B^* \epsilon \mathfrak{B}^*$ and $b \epsilon \mathfrak{B}$, then $B^* + \{b\} \epsilon \mathfrak{B}^*$. For $A' \subseteqq A$ and $b \epsilon B$ let $A'(b)$ be the set of all $a' \epsilon A'$ such that $(a', b) \epsilon R$. Call $\mathfrak{R}$ the set of all ordered pairs (A', B'), where $A' \subseteqq A$, $B' \subseteqq B$, and for each $a' \epsilon A'$ there is at least one $b' \epsilon B'$ such that $a' \epsilon A'(b')$. It is then proved that the two following statements are equivalent: (1) If $A' \subseteqq A$, $B' \subseteqq B$, and $(A', B') \epsilon \mathfrak{R}$, then there exists $B^* \epsilon \mathfrak{B}^*$ such that $B^* \subseteqq B'$, $(A', B^*) \epsilon \mathfrak{R}$. (2) If $A' \subseteqq A$, $A' \notin \mathfrak{A}^*$, then there exists at least one element a' of A' such that $(a', b) \epsilon R$ implies $A'(b) \notin \mathfrak{A}^*$. If A is a space, B a set of subsets of A, R the set of those (a, b) for which a is an interior point of b, $\mathfrak{A}^*$ the set of all subsets of A whose power is $\leqq \mathfrak{n}$ (an infinite cardinal number), $\mathfrak{B}^*$ the set of all sets of $\mathfrak{n}$ or less subsets of A, then one obtains the theorems of Borel, Lebesgue, Young, and Lindelöf. (Received November 19, 1937.)

38. A. N. Milgram: *An existence theorem and some applications to topology.*

A class A of elements $a, a', a'', \cdots$, and a subset T of the cartesian product $A \times A = A^2$ are supposed given; T is such that (1) $(a, a) \epsilon T$ and (2) $(a, a') \epsilon T$, $(a', a'') \epsilon T$ implies $(a, a'') \epsilon T$. In addition one assumes given an arbitrary sequence B of objects $b_1, b_2, \cdots, b_i, \cdots$, and a subset R of $A \times B$ with the property that for any i, if $(a, a') \epsilon T$ and $(a, b_i) \epsilon R$, then $(a', b_i) \epsilon R$. If A' is a subset of A, the element a^* of A' is called an extremal in A' if for each element a' of A' the relation $(a^*, a')\epsilon T$ implies $a^*=a'$. R is called a separator of A' if, for any pair of elements $a'\neq a''$ of A' such that $(a', a'')\epsilon R$, there exists an i for which $(a', b_i)\notin R$ and $(a'', b_i)\epsilon R$. The following theorem is proved: If A' is a subset of A, R a separator of A', and a_0 an element of A', there exists a sequence $a_0, a_1, \cdots, a_n, \cdots$, of elements in A' with $(a_n, a_{n+1})\epsilon T$, $(n=0, 1, 2, \cdots)$, such that whenever a^* is an element of A' and $(a_n, a^*)\epsilon T$ for every n, then a^* is an extremal. Various classical results in different branches of mathematics are special cases of this theorem, as, for example, the Brouwer reduction theorem and certain extreme value theorems in the calculus of variations. (Received November 19, 1937.)

39. Karl Menger and A. N. Milgram: *On linear subsets of metric spaces.*

A subset of a metric space is linear if it is congruent to a subset of E_1 (the euclidean line). The sum of a monotonically increasing sequence of linear subsets of a space is linear. The closure of a linear set is linear. By virtue of a certain saturation theorem, a subset S of a metric space, for any two distinct points p and q, contains a linear subset L_{pq} which is not a proper subset of any other linear subset of S containing p and q. If S is complete, then L_{pq} is complete, thus congruent to a closed subset of E_1. If in addition S is convex, then L_{pq} is convex and thus congruent to a segment, a ray, or E_1 itself. (Received November 19, 1937.)

40. W. E. Milne: *The remainder term for approximations of linear type.*

A large class of formulas, including most of the formulas for polynomial and trigonometric interpolation, formulas obtained by the method of least squares, formulas for numerical differentiation and integration, and formulas such as Hermite's formula for interpolation using both ordinates and slopes, all have the common characteristic of being *linear* with respect to the function to which they are applied. A general expression is obtained for the remainder term for formulas having this property. In a number of important cases this general remainder term is reduced to a form suitable for immediate applications to practical problems. In some instances, for example, Lagrange's interpolation formula and the Newton-Cotes quadrature formulas, the result is found to be identical with well known expressions for the remainder. In other cases some interesting forms have been obtained which are believed to be new. (Received November 13, 1937.)

41. M. G. Moore: *On expansions in series of exponential functions.*

Let $h(t)=\sum_{\mu=1}^{n}c_\mu \exp(\alpha_\mu t)$, where α_μ and c_μ are complex constants. Let P be the smallest convex polygon containing the points α_μ. Then $f(x)$ is expanded in a series of the form $\sum_{s=1}^{\infty}\sum_{k=1}^{\sigma}P_{ks}(x)\exp(t_{ks}x)$, where σ is a function of s, and $P_{ks}(x)$ is a polynomial of degree one less than the order of the zero t_{ks} of $h(t)$. The expansion, in the most general case considered, is convergent in a convex polygon contained in P. The method used is that of application of a generalization (obtained by the author) of the Fourier biorthogonality properties. The Fourier theorem, as stated for a Lebesgue summable function which is of bounded variation in certain neighborhoods, is one of the special cases of the convergence theorem which arises when the polygon P has only two sides. (Received November 18, 1937.)

42. M. G. Moore: *On expansions in solutions of differential equations.*

The author makes a generalization of the properties of biorthogonality, given in a previous paper (see abstract 44-1-41), to include the Birkhoff expansion theory. The term by term difference of the resulting formal expansion and the series of exponentials of the preceding paper is shown, under certain broad hypotheses, to converge uniformly to zero in certain regions. A discussion is made of the Green's functions and adjoint series belonging to the problem. The author applies the results of the paper to obtain properties of uniform convergence for the series of his preceding paper. (Received November 18, 1937.)

43. A. P. Morse: *A continuous function with no unilateral derivatives.*

A. S. Besicovitch (Bulletin de l'Académie des Sciences de Russie, vol. 19 (1925), pp. 527–540) has constructed an even continuous function B for which he asserts the properties $D_+B(x)<D^+B(x)$, $D_-B(x)<D^-B(x)$ at each point x of the interval $(-a, a)$. Later E. D. Pepper (Fundamenta Mathematicae, vol. 12 (1928), pp. 244–253) examined this same function. Besicovitch's construction is geometric and some readers have found his reasoning difficult to follow. S. Saks (Fundamenta Mathematicae, vol. 19 (1932), pp. 211–219) has proved that the "functions of Besicovitch" constitute a set of only first category in the space C of continuous functions. In the present paper there is exhibited a function which at no point of $(-1, 1)$ has a unilateral derivative (finite or infinite); in fact it has, in addition, the property that at each point of $(-1, 1)$ at least one derivative number on each side is infinite. Like Besicovitch the author employs the idea of associating with a function having dense intervals of constancy another such function. The method of association is arithmetic, however, and differs essentially from that used by Besicovitch. (Received November 22, 1937.)

44. S. B. Myers and N. E. Steenrod: *The group of isometries of a Finsler manifold.*

A Finsler manifold is an ordinary n-dimensional manifold of class C^4, provided with an arc-length defined by $\int F(x, x')dt$. The integrand F is to be invariant and of class C^3, positive, positively-regular, and positively-homogeneous of first degree in (x'). The class of Finsler manifolds includes the Riemannian manifolds. A Finsler manifold M becomes a metric space if distance $\rho(A, B)$ is defined as the greatest lower bound of the lengths of arcs AB. An *isometry* of M is a distance-preserving homeomorphism of M into itself. The group of isometries of M can be topologized so that a sequence of isometries T_i converges to an isometry T if and only if $T_i(P)$ converges to $T(P)$ for every P on M. The object of the present paper is to prove that under this topology the group of isometries of a compact Finsler manifold is an abstract Lie group. (Received November 23, 1937.)

45. C. J. Nesbitt: *Relations between the coefficients of the modular representations of groups.*

In the theory of ordinary representations of finite groups, the well known orthogonal relations are fundamental. Extensions of these relations hold for modular representations. Let a prime ideal divisor of a rational prime p be the modulus. Denote by

U a modular indecomposable constituent of the regular representation R of a finite group G; by F, the modular irreducible representation of G which is uniquely associated with U; by f, the degree of F; and by A, the square matrix, of degree f, appearing in the lower left corner of U. If, for the element g of G, $A(g)=(a_{\kappa\lambda}(g))$, $F(g)=(f_{\kappa\lambda}(g))$, it results that (1) $\sum_{g\subset G}a_{\alpha\rho}(g^{-1})f_{\sigma\beta}(g)=c\delta_{\alpha\beta}\cdot\delta_{\rho\sigma}$, where $c\not\equiv 0 \pmod p$; (2) $\sum_{g\subset G}b_{\alpha\rho}(g^{-1})f_{\sigma\beta}(g)=0$, where $b_{\alpha\rho}$ is a coefficient of some indecomposable constituent of R, but is not contained in A. By a related argument it follows that there is no indecomposable representation, other than U itself, which contains U as constituent. U is said to be of the first kind if each of its irreducible constituents is equivalent to F. For such U the trace of A is a class function, and a character relation results from (1). (Received November 23, 1937.)

46. Rufus Oldenburger: *Non-singular linear combinations of general forms.*

Each form $M=a_{ij\ldots m}x_iy_j\cdots z_m=x_iM_i$, $(i=1,\cdots,r)$, can be written in a field F as the sum $S=A_\alpha B_\alpha\cdots D_\alpha$, $(\alpha=1,\cdots,s)$, for some s, where $A_\alpha,\cdots,D_\alpha$ involve the variables $x_i,\cdots,z_m$, respectively. A form is said to be non-singular in F if it has a representation S where the forms in each of the sets $(A_\alpha),\cdots,(D_\alpha)$ are linearly independent. Non-singular forms were studied elsewhere by the author (Transactions of this Society, 1936); it was proved that for each non-singular form M there exist values a_i of x_i in F such that a_iM_i is non-singular. If the forms in each of the sets $(B_\alpha),\cdots,(D_\alpha)$ are linearly independent, M is said to be non-singular in F on $j,\cdots,m$. The principal contribution of this paper is a proof that if M is non-singular in F on $j,\cdots,m$, there exist values a_i of x_i in F such that a_iM_i is non-singular. Using this result, one can prove that a form M of degree $\geqq 4$ is non-singular in F on $j,\cdots,m$ if and only if (a) there exist values a_i of x_i in F such that $H=a_iM_i$ is non-singular; (b) if (a) is satisfied, the x's being so labeled that $a_1\neq 0$ in H, the non-singular linear transformations which bring H into the canonical form $x_1\cdots z_1+\cdots+x_n\cdots z_n$ bring $M_2,\cdots,M_r$ into forms with diagonal matrices. (Received November 20, 1937.)

47. E. W. Paxson: *On the postulates for linear topological spaces.*

Von Neumann has given a useful set of postulates for linear topological spaces (Transactions of this Society, vol. 37 (1935), p. 4). The author shows by simple examples that (2), (4), (5), and (6) (loc. cit., Definition 2b) constitute an independent set, demonstrating that, primarily on the basis of the continuity postulate (4), the Hausdorff intersection axiom (3) is fulfilled. (Received December 3, 1937.)

48. G. B. Price: *The theory of integration.*

This paper considers the integration of functions defined over certain abstract spaces with values in Banach spaces. It extends and unifies the theories of integration developed by Bochner, Garrett Birkhoff, and others, and determines the interrelations and limitations of these various special theories. In order to establish these results, a class of sets is defined and studied which includes convex sets as a special case. The convergence of infinite series whose terms are sets is investigated. Measurable functions are defined, their properties established, and their integrals investigated; the measurable functions of Bochner (Fundamenta Mathematicae, vol. 20 (1933), pp. 262–276) are a special case of the measurable functions here defined. The Riemann-Stieltjes integral is treated, and the Darboux theory is extended to these integrals. Finally, some results are given on singular integrals and Fourier series. (Received November 20, 1937.)

49. G. Y. Rainich: *Dirac equations and conditional invariants.*

Special cases of Dirac equations have been translated into ordinary tensor analysis by Gordon Fuller (abstract 42-11-445), who used certain quadratic combinations of the psi's introduced by Darwin. In connection with some of these special cases, the author introduced conditional invariants (abstract 43-3-180). Early this year, Whittaker introduced another set of quadratic quantities and expressed the Dirac equations in terms of them, using an irrational differential operation. In the present paper, quantities are introduced which are closely related to Whittaker's, but avoid the use of both self-dual six-vectors and the complex conjugate. In terms of these quantities, the Dirac equations are written using ordinary tensor analysis without irrational operations, but employing conditional invariants. In three-dimensional terminology the situation may be described as follows: consider three mutually perpendicular vectors of equal length; two of them may be interpreted as electric and magnetic vectors, the third as the momentum vector of matter, their common length as density; then the left sides of the equations are the left sides of Maxwell's equations but the right sides, instead of being zero, are components of a certain pair of singular conditional covariant vectors. (Received November 23, 1937.)

50. W. C. Randels: *On the absolute summability of Fourier series.*

An example is given of a function whose Fourier series is absolutely summable $|A|$ but not $|C_\alpha|$ for any α. (Received November 3, 1937.)

51. W. T. Reid: *A theorem on quadratic forms.*

In this note, the following theorem is proved: If $A[x]$ and $B[x]$ are real quadratic forms in (x_α), $(\alpha=1, \cdots, n)$, and $A[x]>0$ for all real $(x_\alpha)\neq(0_\alpha)$ satisfying $B[x]=0$, then there exists a constant λ_0 such that $A[x]-\lambda_0 B[x]$ is a positive definite form. This result is of use in considering the Clebsch condition for multiple integrals of the calculus of variations. Recently, A. A. Albert (see abstract 43-11-395) has obtained an algebraic proof of this result. The proof of this note is more analytic in character than that of Albert, and is of interest because in the proof itself there is obtained directly, in terms of the roots of the associated characteristic equation, the interval in which the value λ_0 of the theorem must be chosen. (Received November 23, 1937.)

52. R. F. Rinehart: *Commutative algebras which are polynomial algebras.*

By a polynomial algebra P modulo $p(x)$ is meant the algebra of the residue classes of the ring of all polynomials with coefficients in a field F, modulo the polynomial $p(x)$ with coefficients in F. If $p(x)$ is irreducible, then P is a field. In this paper a study is made of the case where $p(x)$ is not necessarily irreducible. If $p(x)$ is reducible, then P is a direct sum of primary polynomial algebras whose moduli are the powers of the distinct irreducible factors of $p(x)$, and, conversely, a direct sum of polynomial algebras is a polynomial algebra. This fact is used to determine the structure of commutative algebras which are equivalent to polynomial algebras. (Received November 23, 1937.)

53. W. C. Risselman: *On periodic continued fractions.*

Infinite periodic continued fractions of the Stieltjes-Grommer type are under consideration. Trigonometric forms for the numerators and denominators of the convergents are used to study the sequence of convergents on the intervals of the real axis on which the discriminant of the associated quadratic equation is non-positive. (Received November 26, 1937.)

54. J. B. Rosser: *On the consistency of Quine's "New foundations for mathematical logic."*

The author has investigated the consistency of the system of logic presented by Quine in *New foundations for mathematical logic* (American Mathematical Monthly, vol. 44 (1937), pp. 70–80). This was reviewed by Bernays in the Journal of Symbolic Logic, vol. 2 (1937), pp. 86–87. As Bernays pointed out in his review, the system is inconsistent if one of Quine's definitions of stratification be used. Using the other definition of stratification, an attempt has been made to obtain the usual contradictions, but without success. The reasons for the failure of the particular attempts are explained. In attempting to find ω-inconsistencies or inconsistencies of a more subtle character, an inferential rule of a type due to Kleene was added to the system. This rule has the property that the system will not contain any inconsistencies after addition of the rule unless it contained inconsistencies (possibly of a very indirect sort) before addition of the rule. This rule makes possible the proof of a number of interesting results, among them the axiom of infinity, but apparently gives no inconsistencies. (Received November 20, 1937.)

55. J. B. Rosser and R. J. Walker: *On the transformation group for diabolic magic squares of order four.*

A group of transformations is given by which any diabolic magic square of order four is transformed into another such square. Furthermore every such square can be derived from a single normalized square by a unique transformation of the group. Generators of the group are given and the group is shown to be of order 384. Hence there are exactly 384 diabolic magic squares of order four. (Received November 17, 1937.)

56. E. H. Rothe: *Theory of the topological order in linear topological spaces.*

As was shown in a former paper to appear in Compositio Mathematica, the topological theory of the order of a point with respect to the image of a sphere holds in Banach space. It is now shown that the theory holds also in certain non-metric spaces. (Received November 22, 1937.)

57. O. F. G. Schilling: *Algebraic theory of abelian functions.*

Let K be a function field of genus p whose field of constants is an arbitrary, algebraically closed field. The author generalizes the classical theory of abelian functions. He considers the algebraic variety of all unordered p-tuples of points on K and develops on this manifold the analogue of the classical theory. Using van der Waerden's notion of generic point one sees that the rational functions of this variety form a field A of p variables which is given as the subfield of the p-fold direct product of

K with itself. By applying the theory of algebraic correspondences, it is proved that A and its associated variety admit the group G of all divisor classes of degree 0 in K as a group of birational transformations. The group of all birational transformations on A contains the reflection. The group G_n of all divisor classes of order n is a finite group for each n; G_n is the Galois group of A over the field A_n of natural multiplications of A by n; and A is unramified over A_n. There exists also an equivalent for the complex multiplications. All these results will be published in a series of papers, partially in collaboration with H. Busemann. (Received November 19, 1937.)

58. O. F. G. Schilling: *A note on infinite perfect fields.*

Let k be a field which is perfect with respect to a continuous valuation of rank one and let κ be its field of residue classes. If k' denotes a subfield of k which is dense in k, in the sense of the given valuation, and whose field of residue classes coincides with κ, then it is shown that each normal extension $k(A)$ of k can be obtained as the join of a suitable superfield $k'(A')$ of k' by means of k: $k(A)=k'(A')k=k(A')$. This theorem is of importance for the investigation of the structural theory of general infinite perfect fields. (Received November 12, 1937.)

59. O. F. G. Schilling: *On the structure of local class field theory.*

Let k be a field which is perfect with respect to a discrete valuation of rank one and whose field of residue classes κ is a finite Galois field. The theory of finite abelian extensions and normal algebras over k was developed by H. Hasse. The author investigates problems of the following type: given a perfect field k and a theorem of Hasse's theory as axiom for k, what can be said about the structure of k? It is shown that the more significant theorems taken as axioms for k imply that, for each integer n, there exists exactly one cyclic unramified extension of k. In order to obtain the most general p-adic fields, and x-adic fields, where the residue fields κ are infinite Galois fields which admit extensions of any degree n, one has to introduce another axiom, namely, that the assumption concerning the validity of a theorem is to hold also for all perfect subfields k' of k whose residue class fields κ' are subfields of κ. This theory is in a certain sense an algebraico-arithmetical equivalent to N. Jacobson's topological theory of perfect fields. However, the class of perfect fields which is described is larger than his. (Received November 12, 1937.)

60. I. J. Schoenberg: *Metric spaces and positive definite functions.*

Let $\mathfrak{S}$ be a semi-metric space with the distance function $\overline{PP'}$. A real continuous even function $g(t)$ is said to be positive definite (p.d.) in $\mathfrak{S}$ if $\sum_1^n g(\overline{P_iP_k})\rho_i\rho_k \geqq 0$, for arbitrary real ρ_ν, any n points P_ν of $\mathfrak{S}$, and this for $n=2, 3, 4, \cdots$. Thus $g(t)=e^{-\lambda t^2}$, $(\lambda>0)$, is p.d. in Hilbert space $\mathfrak{H}$. As Theorem 1, it is proved that a semimetric space $\mathfrak{S}$ is isometrically imbeddable in $\mathfrak{H}$ if and only if $\mathfrak{S}$ is separable and all functions of the family $e^{-\lambda t^2}$, $(\lambda>0)$, are p.d. in $\mathfrak{S}$. Let $\mathfrak{S}(\gamma)$ denote the new space obtained from $\mathfrak{S}$ if the metric $\overline{PP'}$ is replaced by $(\overline{PP'})^\gamma$, $(\gamma>0)$. A new simple proof is given of the author's previous result: $\mathfrak{H}(\gamma)$, $(0<\gamma<1)$, is isometrically imbeddable in $\mathfrak{H}$. Theorem 1, together with previous results on screw lines in $\mathfrak{H}$ by von Neumann and the author, leads to the following purely analytical result in the theory of p.d. functions: The most general positive function $f(x)$ whose positive powers $[f(x)]^\lambda$, $(\lambda>0)$, are all positive definite is of the form $f(x)=\exp\{c-\int_1^\infty(\sin xu)^2/u^2 d\sigma(u)\}$, where $\sigma(u)$ is non-decreasing for $u\geqq 0$ such that $\int_1^\infty u^{-2}d\sigma(u)$ is finite, while c is a real constant. A further consequence of Theorem 1 is as follows: If $0<p\leqq 2$, then $L_p(\gamma)$, $(0<\gamma\leqq p/2)$, is isometrically imbeddable in $\mathfrak{H}$. For $p=2$ we have the previous result on $\mathfrak{H}(\gamma)$. Similar statements for $p>2$ depend on yet unsolved problems in the theory of p.d. functions of several variables. (Received November 22, 1937.)

61. W. E. Sewell: *Jackson summation of the Faber development.*

Let C be an analytic Jordan curve in the z-plane. Let $f(z)$ be analytic in C, continuous in $\overline{C}$, and let $f^{(p)}(z)$, $p\geqq 0$, satisfy a Lipschitz condition of order α, $0<\alpha\leqq 1$, on C. Then one has $\left|f(z)-\sum_{\nu=0}^{n} d_{n\nu}a_\nu P_\nu(z)\right| \leqq M/n^{p+\alpha}$, z in $\overline{C}$, where M is a constant independent of n and z, $\sum_{\nu=0}^{n} a_\nu P_\nu(z)$ is the sum of the first $n+1$ terms of the development of $f(z)$ in the Faber polynomials belonging to C, and $d_{n\nu}$ is the Jackson summation coefficient. (Received November 16, 1937.)

62. H. A. Simmons: *Affine differential geometry of plane curves from the point of view of Wilczynski.*

This paper contains: certain comparisons between Blaschke's method of studying the affine differential geometry of plane curves and a corresponding method which Wilczynski suggested to the author; an affine analogue of a general theorem of E. P. Lane on the differential equation which is appropriate for use in Wilczynski's projective differential geometry of curves in a linear space of $n\geqq 2$ dimensions; application of the Lie theory of differential equations to distinguish sharply between the affine and the projective differential geometries of plane curves from Wilczynski's point of view; and analogs of most of the results of Wilczynski on projective differential geometry of plane curves. Special results of interest in this paper are: equations of the osculating conic and of the osculating cubic, coordinates of the Halphen point, and a proof of the fact that in the affine differential geometry of plane curves a curve may be anharmonic when the coefficients of the associated differential equation are not all constants. (Received November 15, 1937.)

63. M. F. Smiley and L. R. Wilcox: *Metric lattices.*

In the theory of continuous geometries a real-valued dimension function $D(a)$ defined over a lattice L and satisfying $D(a+b)+D(ab)=D(a)+D(b)$, $D(c)<D(d)$ for $c<d$, is shown to give rise to a metric $\delta(a, b)=D(a+b)-D(ab)$. This topology and its relation to that determined by the partially ordering relation have been studied by J. von Neumann (Proceedings of the National Academy of Sciences, vol. 22 (1936), p. 106). In a study of affine geometry by means of the theory of lattices it is shown that dimension functions satisfy $D(a+b)+D(ab)\leqq D(a)+D(b)$ (K. Menger, Annals of Mathematics, vol. 37 (1936), pp. 465–466). The present paper deals with lattices in which dimension functions satisfying conditions weaker than this latter condition are defined. A metric topology may be introduced by defining $\delta(a, b)\equiv 2D(a+b)-D(a)-D(b)$; a characterization is given for metrics arising in this manner. Certain interrelations between this topology and the order topology are investigated; a typical result is that, under several mild hypotheses, completeness of L in the metric implies that L is continuous. (Received November 22, 1937.)

64. I. S. Sokolnikoff and E. S. Sokolnikoff: *Torsion of regions bounded by circular arcs.*

The torsion problem requiring a harmonic function to assume the value $(x^2+y^2)/2$ on the boundary of some finite regions, bounded by pairs of circular arcs, is solved in closed form. The method used is to map the regions in question upon a unit circle and evaluate some integrals equivalent to those of Schwarz. (Received November 19, 1937.)

65. Daisy M. Starkey: *On a fiducial test of the significance of the difference of the means of two normally distributed populations which are not known to have equal variances.*

Essentially the problem consists of finding the distribution of $at+bt'$, where a and b are constants, and t and t' are distributed in Student's distribution. This paper represents an attempt to generalize the recent work of R. A. Fisher concerning samples of two, and considers: (i) The case in which the sample numbers N, N' are even and small. An expression for the characteristic function of Student's distribution valid for all even values of N is used, and, if $N=N'$, the distribution and probability integral of θ are derived, where $(N-1)^{1/2}\tan\theta=(at+bt')/(|a|+|b|)$. The population means may be said to differ significantly if the quantity $\theta_1=\text{arc tan }(\bar{x}-\bar{x}')/n^{1/2}(s+s')$ is significantly large, where $s^2=\sum(x-\bar{x})^2/N(N-1)$, and $n=N-1$. (ii) The case in which N and N' are large and of the same order of magnitude. R. A. Fisher's asymptotic expansion of Student's distribution is used, and, if $N=N'$, an asymptotic expansion for the distribution of $(at+bt')/(a^2+b^2)^{1/2}$ in terms of n is derived, and also for the probability integral, in which the quantity $(\bar{x}-\bar{x}')/(s^2+s'^2)^{1/2}$ may be tested. If the ratio of the population variances were known, the exact distributions of $(\bar{x}-\bar{x}')/n^{1/2}(s+s')$ and $(\bar{x}-\bar{x}')/(s^2+s'^2)^{1/2}$ should be used. (Received November 30, 1937.)

66. Eugene Stephens: *The purely operational solution of linear differential equations with polynomial coefficients.*

By the transformation $x=e^z$ and the operational identity $x^nD^n\equiv\prod_{\alpha=0}^{n-1}(\theta-\alpha)$, equations of the form $\sum_k F_{hk}(s)\cdot D^k\cdot y=Q(x)$, with $F_{hk}=\sum_h C_h x^h$, become equations of the form $\sum_m f_m(\theta)\cdot e^{mz}\cdot y=Q(e^z)$. Division by any one of the $f_m(\theta)$ gives a form $[1+\sum_i\phi_i]\cdot y=P(z)$, the operational solution of which is $y=[1+\sum_i\phi_i]^{-1}\cdot[P(z)+0]$. A reciprocal theorem which transforms the inverse operator $[1+\sum_i\phi_i]^{-1}$ into a series of direct operators is then applied. Two reciprocal power series in x, one with the coefficients ϕ_i and the other with the undetermined coefficients F_k as the direct operators, are multiplied together and the coefficients equated to obtain the relations between the ϕ_i and the direct operators F_k. As many series solutions may be obtained as there are terms $f_m(\theta)$ in the (z, θ) form, and each of these has as many arbitrary constants as the degree in θ of the respective $f_m(\theta)$ used as a divisor. Systems are transformed in the same manner, but the eliminant of the system is obtained by using the determinant of the matrix of the (z, θ) operator coefficients as the operator on an unknown function. The individual unknowns are then proportional to the cofactors of any row in the matrix of coefficients, the function of proportionality being the solution of the eliminant equation. Regular and singular systems are considered. (Received November 23, 1937.)

67. Otto Szász: *Some inequalities in the theory of Fourier series.*

Suppose $f(x)$ is real or complex valued, periodic of period 2π, and Lebesgue integrable on $(-\pi, \pi)$. Denote its Fourier series by $f(x)\sim\sum_{-\infty}^{\infty}c_\nu e^{i\nu x}$, and let $P_n(z)=\sum_1^n\nu c_\nu z^\nu+\sum_{n+1}^{2n}(2n-\nu)c_\nu z^\nu$. It is shown that if $|f(x)|\leqq\mu$ for all x, then $|P_n(z)|/n\leqq\mu$, for $|z|\leqq 1$, $n=1, 2, 3, \cdots$. If $f(x)$ is real, then also $|\mathfrak{R}P_n(e^{ix})|/n\leqq 2\mu/\pi$,

$|\Im P_n(e^{ix})|/n \leqq 2\mu/\pi$, $n=1, 2, \cdots$. The cases where equality occurs in the above are also considered. The last inequality for $x=0$ leads to an improvement of an estimate given by the author in the American Journal of Mathematics, vol. 59 (1937), p. 701. (Received November 19, 1937.)

68. H. S. Thurston: *Matric conjugates in a ring $R(A)$.*

If e_i, $(i=1, 2, \cdots, n)$, are the principal idempotent elements of a square matrix A having distinct latent roots α_i, it is well known that (a) any matrix $M=f(A)$ in the ring $R(A)$ has latent roots $\mu_i=f(\alpha_i)$ and (b) $M=\sum \mu_i e_i$, an equality which may be symbolically expressed as $M=(\mu_1, \mu_2, \cdots, \mu_n)$. This symbolism provides an exceptionally convenient way of determining sets of matrices conjugate to M, of the types considered by Taber, Franklin, Sokolnikoff, and Hermann. Sets of conjugates belonging to none of these types are also obtained. (Received November 20, 1937.)

69. C. B. Tompkins (National Research Fellow): *Deformations of the inner equator of a torus.*

This paper shows that if the inner equator of a torus is continuously deformed completely around the surface and back onto itself, the maximum length necessary is that of the closed geodesic of the same topological type as the equator and intersecting the outer equator in diametrically opposite points; furthermore, there is no deformation which does not contain a curve of at least this length. This is a result suggested by Marston Morse. That the length suffices is shown by a simple example; the necessity of including a curve of at least this length is shown by proving that some curve of the family passes through diametrically opposite points of the outer equator. This last is a corollary of the proposition: If A, B, and C are concentric circles in a plane, A inside B which is inside C, any continuous deformation of A into C passes through some curve which intersects B in diametrically opposite points. (Received November 23, 1937.)

70. W. J. Trjitzinsky: *Theory of functions of a complex variable defined over general sets.*

The author develops a theory of functions $f(z)=u(x, y)+iv(x, y)$ of a variable $z=x+iy$, possessing unique first derivatives on a perfect set E. More precisely, he studies "general monogenic" functions for which u, v, the first partials of u and v, and the second partials of u are continuous in E. Here u is uniform and $\Delta u=0$ in E, (Δ is the Laplacian operator), while v is a harmonic conjugate in E of u. It is established that such functions $f(\alpha)$ are representable in E as $h(\alpha)+\int\int q(x, y) \log (z-\alpha) dxdy$. Here $h(\alpha)$ is analytic and $q(x, y)$ is real continuous, $q(x, y)=0$ in $C(E)$ (the complement of E). The set $C(E)$ is covered by a set of domains $|z-A_i|<\gamma_i$, $(i=1, 2, \cdots)$. An extensive investigation is made on the basis of the principles according to which the faster $q(x, y)\rightarrow 0$ (as $z\rightarrow$the frontier of $C(E)$) and the faster $\gamma_i\rightarrow 0$ (as $i\rightarrow\infty$) the more regularity properties (existence of derivatives, various representations, various uniqueness properties) will the corresponding g.m. functions possess. The work will appear (in French) in the Annales de l'École Normale Supérieure. (Received November 22, 1937.)

71. A. W. Tucker: *Symmetric and alternating products of circles.*

The following results are established by quite elementary methods: (1) the symmetric product of $2n+1$ circles is homeomorphic with the direct topological product of a $2n$-cell and a circle, (2) the symmetric product of $2n$ circles is homeomorphic with a $2n$-dimensional "Möbius band," and (3) the alternating product of $n+1$ circles is homeomorphic with the direct topological product of an n-sphere and a circle. (Received November 24, 1937.)

72. H. E. Vaughan: *On the characterization of abstract spaces by postulating the existence or non-existence of certain types of metrics.*

Let E be a metrisable space, M the class of all metrics consistent with the topology of E. Then M contains the following subclasses: B, C, TB, and TC, consisting of all metrics in which E is bounded, complete, totally bounded, and totally complete, respectively. Using these concepts as principles of classification, M can be expressed as the sum of seven disjoint sets, which generate a Boolean algebra with 2^7 elements, in general distinct. This paper investigates all cases in which this algebra degenerates into algebras with fewer elements, thus obtaining systematically all theorems of the type suggested in the title. (Received November 23, 1937.)

73. H. S. Wall: *Abelian hypergroups.* Preliminary report.

A set A is an abelian hypergroup if: (1) $a+b=[c]$ for $a\varepsilon A$, $b\varepsilon A$, where $[c]$ is a finite or infinite subset of A; (2) $a+b=b+a$; (3) $a+(b+c)=(a+b)+c$; there is a $0\varepsilon A$ such that if $a\varepsilon A$, $a+0=a$; (5) $a\varepsilon A$ implies that there exists uniquely $-a\varepsilon A$ such that $0\varepsilon a=a$; (6) $a+b=[c]$ implies $-a-b=[-c]$. The hypergroup A is homomorphic with A' if there exists a function f such that if $a\varepsilon A, f(a)\varepsilon A'$; $f(0)=0'$; and $f(a)+f(b)=[f(c)]$ where $a+b=[c]$. A "hyperfield" of these operators f has the property that multiplication is single-valued and addition is many-valued. Difference hypergroups may be formed, and if $A=A'$ there exists the isomorphism $A-f(A)\approx g(A)-g(A)\cap f(A)$, where g is any one of the "branches" of $1-f$. The separate branches of $f(a)+g(a)$ enjoy a sort of "continuity" in a certain subhypergroup containing $a\varepsilon A$. The Jordan-Hölder theorem, and the notions of linear dependence and rank can be extended to A; and topological systems of abelian hypergroups can be defined to which may be carried the formal theory of topological systems of abelian groups. (Received November 17, 1937.)

74. S. E. Warschawski: *On the degree of approximation in some convergence theorems in conformal mapping.*

Let $w=f(z)$ map the circle $|z|<1$ conformally onto the interior R of a closed Jordan curve C, $(f(0)=w_0$, w_0 in R, and $f'(0)>0)$. It is known that, under a suitable continuous deformation of $C, f(z)$ and $f'(z)$ vary continuously with C. In the present paper estimates for the degree of the variation of $f(z)$ and $f'(z)$ are obtained. (1) Suppose: (a) C lies in the annular region $1-\epsilon\leqq|w|\leqq 1+\epsilon$, and (b) C has continuously turning tangent $\theta(s)$, $(\theta(s)=$inclination angle, $s=$arc length), and $|\Delta\theta/\Delta s|\leqq 1+\epsilon$. If $w_0=f(0)=0$, $f'(0)>0$, then there is an absolute constant M such that, in $|z|\leqq 1$, $|f'(z)-1|\leqq M\epsilon$. (2) Suppose: (a) C and C^* are two closed Jordan curves such that C^* lies in the region G_ϵ, obtained when a circle of radius ϵ is described about every point of C; (b) C and C^* have continuous curvature $\kappa(s)$ and $\kappa^*(s^*)$ respectively, and, if P^* is a point on C^* within the circle of radius ϵ about P of C, then $|d\kappa/ds(P)-d\kappa^*/ds^*(P^*)|\leqq\epsilon$; (c) C and C^* lie within a circle $|w-w_0|=R$ and contain a circle $|w-w_0|=r$ in their interiors. If $f(z)$ and $f^*(z)$ map $|z|<1$ onto the interior of C and C^* respectively and if $f(0)=f^*(0)=w_0$, $f'(0)>0$, $f^*(0)>0$, then $|f(z)-f^*(z)|\leqq M\epsilon$, where M is a constant depending only upon r and R. (3) Some weaker estimates are obtained under weaker hypotheses. (Received November 23, 1937.)

75. D. L. Webb: *A set of postulates for an algebra of n-valued logic.*

All of the elements of Boolean algebra can be expressed in terms of elements which E. V. Huntington (Transactions of this Society, vol. 5 (1904), pp. 308, 309) calls irreducible elements and B. A. Bernstein (American Journal of Mathematics, vol. 57 (1935), pp. 733–742) minimal elements. A set of independent postulates is found for the generalization of these elements that occur in the algebra of n-valued logic and for their combination by means of an operation of addition. The algebra of n-valued logic is defined as consisting of all possible sums of these irreducible elements. All algebras, for a given n, and with the same number of elements, are found to be simply isomorphic. In the case for $n=2$, the algebra becomes a Boolean algebra. (Received November 20, 1937.)

76. G. T. Whyburn: *Interior transformations on certain curves.*

Let $T(A)=B$ be an interior transformation where A is compact. In this paper the following results are established: (1) If D is any dendrite in B there exists a dendrite E in A which maps topologically onto D under T. If A is a boundary curve (that is, a locally connected continuum every true cyclic element of which is a simple closed curve), so also is B. (3) If the boundary of every connected open subset of A is totally disconnected, then T is necessarily light, that is, dim $T^{-1}(b)=0$ for every $b\varepsilon B$. (4) If A is a dendrite, then (a) for every $x\varepsilon B$, $T^{-1}(x)$ contains at most a finite number of cut points of A, and (b) if for each $x\varepsilon B$ one lets $k(x)$ be the number of points in $T^{-1}(x)$, then for any two points a and b of B (which is necessarily a dendrite) and any point x on the arc ab of B, one has $k(x)\leqq k(a)+k(b)-1$. Finally, an example is given of an interior transformation of a planar graph A onto a non-planar graph B. (Received November 17, 1937.)

77. G. T. Whyburn: *Interior transformations on 2-dimensional manifolds.*

In this paper it is shown that the image of a 2-dimensional manifold (with or without boundaries) under any interior light transformation (that is, a transformation mapping open sets into open sets and mapping no continuum into a point) is itself a 2-dimensional manifold. In particular, any interior light image of a sphere is necessarily a sphere, a projective plane, or a 2-cell. By factoring the general interior transformation into a monotone one and a light interior one, an analysis of the general interior transformation on 2-dimensional manifolds is made possible. (Received November 3, 1937.)

78. W. M. Whyburn: *On the one dimensional Green's function.*

This paper studies the Green's function for nth order linear differential systems with boundary conditions of integral type or which involve a finite or convergent infinite sequence of points of the interval. The familiar method of variation of parameters is used as a starting point in the development of the Green's matrix and in the study of its properties. The results of the paper are used as a guide to a study of new types of boundary value problems for partial differential equations. (Received November 12, 1937.)

79. L. R. Wilcox: *Modularity in the theory of lattices.*

The author has investigated lattices called *affine lattices* which arise from modular lattices by deletion of certain elements (abstract 43-3-159). The present work begins an axiomatic study of lattices possessing some of the properties of affine lattices. The rôle of the modular axiom in the usual theory of independence is studied in detail; a theory of independence for non-modular lattices is obtained by means of an investigation of the closure properties of the set of all pairs (b, c) satisfying $(a+b)c=a+bc$ for every $a\leqq c$. A well known result concerning the equivalence of ascending and descending chain conditions for complemented modular lattices is generalized to a considerably wider class of lattices. Finally, it is shown that the essential results of K. Menger (Annals of Mathematics, vol. 37 (1936), pp. 464–466) for affine geometry hold without assumption of the axiom of complementation, the methods used being an extension of those of Dedekind for modular lattices. (Received November 22, 1937.)

80. Margarete C. Wolf: *The transformation of bases of relative linear sets.*

The definitions of linear extension and linear independence relative to a matrix are those introduced in *Relative linear sets and similarity of matrices whose elements belong to a division algebra*, by M. H. Ingraham and M. C. Wolf, Transactions of this Society, vol. 42 (1937), pp. 16–31. The basic number system is a field F. If $L_M(\xi_1, \xi_2, \cdots, \xi_k)$ is the linear extension, relative to the matrix M, of vectors $\xi_1, \xi_2, \cdots, \xi_k$ which are linearly independent relative to M, and if for a set of polynomials f_{ij} in F, $\eta_i=\sum_{j=1}^{k} f_{ij}(M)\xi_j$, necessary and sufficient conditions that the set η_i, $(i=1, 2, \cdots, l)$, be a linearly independent base relative to M for $L_M(\xi_1, \xi_2, \cdots, \xi_k)$ are obtained for the general problem from cases in which the minimum polynomial g_i such that $g_i(M)\xi_i=0$ is g^{t_i}, a power of an irreducible polynomial g. Necessary and sufficient conditions that such η_i form a linearly independent base relative to M are: (1) for every g^{t_i} there is one and only one η_j such that g^{t_i} is minimum for $g^{t_i}(M)\eta_j=0$; (2) the determinant of the matrix (f_{ij}) is not zero in the ring of polynomials reduced modulo g. The problem is being studied if F is a division algebra, not necessarily commutative. (Received November 19, 1937.)

81. J. M. Feld: *On certain groups of birational contact transformations.*

L. Autonne studied finite linear and quadratic groups of birational contact transformations (crémoniens) in S_2 (Journal de Mathématiques, (4), vol. 3 (1887) and vol. 4 (1888)). In this paper an infinite mixed group Γ is constructed. Γ has an invariant abelian subgroup G. The members of Γ leave a two-parameter family of triangular-symmetric curves invariant. Contragradient variables are used to represent line elements. If (x, u) and (y, v) are a pair of corresponding elements with respect to any member of Γ, the cross-ratio of u and the three lines joining x to the vertices of the fundamental triangle equals the cross-ratio of v and the lines joining y to the same vertices. The dual of this property is also possessed by the transformations. The group Γ has an analogue in S_n. (Received December 28, 1937.)

82. Aaron Fialkow: *Conformal geodesics in Riemann spaces.*

The images $\bar{G}$ in $\bar{V}_n$ of the geodesics of V_n under a conformal transformation $d\bar{s}=\lambda ds$ are called *conformal geodesics* of $\bar{V}_n$ and similarly for the curves G in V_n.

A geometric characterization of conformal geodesics in special spaces was given by Kasner and Lipke, who considered these curves from another viewpoint (dynamics). A simpler derivation of these characteristic properties which has wider validity is given. At each point of $\bar{V}_n$, the principal directions are determined by the tensor $\lambda, ij-\lambda, i\lambda_j$. These are the directions in which the osculating geodesic circles of $\bar{G}$ have higher than second order contact with $\bar{G}$. A similar definiton holds for V_n. The hyperosculating directions for G and $\bar{G}$ correspond. If $\bar{V}_n$ is an Einstein space, the hyperosculating directions for G are the Ricci principal directions of V_n. A characterization of the conformal geodesics of a space of constant curvature as well as properties of the conformal geodesics of an Einstein space are given. The mapping of V_n on $\bar{V}_n$ induces a conformal correspondence on the subspaces of each. Simple relationships are found between the conformal geodesics of V_n and of its subspaces. (Received December 27, 1937.)

83. J. W. Green: *The representation of the solution of the generalized Dirichlet problem.*

If T is a bounded domain and $\phi(\gamma)$ a pseudo-continuous function of its boundary elements (see Perkins, Transactions of this Society, vol. 38, no. 1 (1935)), Perkins has shown that there exists a function $u(P)$, harmonic in T, assuming the values $\phi(\gamma)$ continuously at all pseudo-regular boundary elements. The author defines on the boundary elements a positive additive set function $m(\epsilon, P)$, the weak limit of mass distributions on boundaries approximating to that of T. With the aid of this function it is possible to represent $u(P)$ as a Stieltjes integral $\int_t\phi(\gamma)dm(\epsilon_\gamma, P)$. (Received December 21, 1937.)

84. E. V. Huntington: *A rating table for card-matching experiments.*

Suppose all possible runs with a pack of 25 cards composed of 5 suits of 5 cards each are matched against any fixed run taken from a duplicate pack. The first four moments of the distribution of scores are known. (See T. E. Sterne, Science, Nov. 26, 1937; also E. G. Olds, abstract 43-11-428.) Hence the first four moments of the distribution of the average scores obtained in a sequence of n runs can be computed, and a Charlier curve fitted, for each value of n. Hence a "rating table" is computed, giving, for each value of n, the size of the average score which will not be exceeded more than once in so often. For example, in the case of 4 runs, an average score of 7.50 (or 8.25 or 9.25) will not be exceeded more than once in 75 (or 500 or 10,000) times, respectively. In the case of 100 runs, an average score of 5.46 (or 5.60 or 5.77) will not be exceeded more than once in 75 (or 500 or 10,000) times, respectively. The paper will appear in the forthcoming number of the Journal of Parapsychology (Duke University). (Received December 21, 1937.)

85. E. R. Lorch: *On a calculus of operators in reflexive vector spaces.*

A theory of projections is developed in reflexive Banach spaces, $\mathfrak{B}$ (if the adjoint space is denoted by $\bar{\mathfrak{B}}$, the reflexive character is defined by $\bar{\bar{\mathfrak{B}}}=\mathfrak{B}$). It is demonstrated that projection limits of monotone sequences, projection least upper and greatest lower bounds exist for sets of projections satisfying reasonable assumptions. A theory of projection measure, called resolutions of the identity, is developed for certain completely ordered sets. This theory of measure in turn leads to an operational calculus which establishes the existence of a far-reaching homomorphism between (projection) measurable functions and certain rings of operators in $\mathfrak{B}$. This calculus possesses all the properties attached to that already existing for the special case in which $\mathfrak{B}$ is a Hilbert space, which indicates that the above definition of resolutions of the identity is sufficiently restrictive. (Received December 30, 1937.)

86. E. J. McShane: *Some existence theorems in the calculus of variations. I. Free problems.*

In establishing existence theorems for variation problems requiring $\mathcal{J}(C)$ to be made a minimum, where C is a curve $x=x(t)$ (that is, $x^i=x^i(t)$, $i=1, \cdots, q$) and $\mathcal{J}(C)=\int F(x, x')dt$, the method based on semi-continuity of $\mathcal{J}(C)$ has produced strong results. For isoperimetric problems, however, in which one seeks to minimize $\mathcal{J}(C)$ while holding another integral $G(C)$ constant, that method has been much less fruitful. In this paper an existence theorem for the problem of minimizing $\mathcal{J}(C)$ is established by a method more readily extensible to isoperimetric problems. For each integer n the author finds first a polygon Π_n of not more than n sides which minimizes $\mathcal{J}(C)$ in that class of polygons. The well known properties of minimizing curves for $\mathcal{J}(C)$ (for example, the Weierstrass-Erdmann corner condition) are approximately true for the polygons Π_n. From the restrictions on Π_n thus introduced, one is able, under suitable hypotheses, to show that there is a subsequence of the Π_n which tends both in position and direction to a limit curve C; and it follows at once that $\mathcal{J}(C)=\lim \mathcal{J}(\Pi_n)$, which is the minimum of $\mathcal{J}(C)$. The theorem applies even to certain integrals which are not quasi-regular. (Received December 30, 1937.)

87. F. J. Murray: *Bilinear transformation in Hilbert space.*

A bilinear transformation, $F(f, g)$, is a function of two variables in Hilbert space, linear in f and in g. The elementary theory of linear transformations, with certain modifications, may be extended to $F(f, g)$. For instance, if $F(f, g)$ is continuous at a single point, it is limited, that is, there exists a C such that $\|F(f, g)\| \leqq C\|f\|\cdot\|g\|$. Also if $F(f, g)$ is defined for every pair and is closed, it is limited. A bilinear transformation, $F(f, g)$, determines a linear transformation T from $\mathfrak{H}\otimes\mathfrak{H}$ (F. J. Murray and J. von Neumann, Annals of Mathematics, vol. 37, pp. 116–229) to $\mathfrak{H}$; also any such T determines an $F(f, g)$. If T is closable, then the analysis of linear transformations between Hilbert spaces applies (F. J. Murray, Transactions of this Society, vol. 37, pp. 301–338). If $F(f, F(g, h))=F(F(f, g), h)$ and certain other conditions hold, the study of $F(f, g)$ is shown to depend on the theory of rings of operators (cf. first reference above). If one also has $F(f, g)=F(g, f)$, then, for a suitable realization of $\mathfrak{H}$, $F(\phi(\lambda), \psi(\lambda))=\phi(\lambda)\psi(\lambda)$. Some examples are also studied. (Received December 20, 1937.)

88. C. C. Torrance: *An elementary derivation of the Uspensky quadrature formulas.*

In this note the Newton-Cotes and Uspensky quadrature formulas are derived, in a brief and elementary manner, by merely integrating Taylor's formula with a remainder. No reference is made to any interpolation polynomial. By changing constants of integration, a whole new class of quadrature formulas may be obtained. Generalizations of the Bernoulli polynomials are indicated. (Received December 24, 1937.)

89. Audrey Wishard: *Some conditions on functions of bounded types.* Preliminary report.

The background of this paper is the work of Rolf Nevanlinna on functions of bounded type ("beschränktartige Funktionen"). Necessary and sufficient conditions are found that a function, analytic in the right half plane, be of bounded type there. Three types of conditions are found. They are expressed in terms of the behavior of certain integrals along vertical, horizontal, or radial straight lines, rather than along the semi-circles used by Nevanlinna. The properties of the integrals are studied in an effort to determine the asymptotic behavior of the average values of functions of bounded type. (Received December 18, 1937.)